classmate
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Module :-1 Theory of equation - Introduction to Business mathematics and its importance. equation, meaning, Types of equation. Simple or linear equation, simultaneous equation [only two variables], elimation and substitution method Quadratic equation. factorzation and formulae method $(Ax^2 + bx + c = 0)$. Simple problems.

Module :2 Indices, Matrices and logarithms - Meaning, types operation on matrices, addition, subtration and multiplication of 2 matrices, transpose, determinants. minor of element, Co-factor of element, inverse, crominants into two variable, problems
Indices - Meaning, basic laws of indices and there application for simplification.
logarithms - Laws of logarithm, common logarithm application of log table for simplication.

Module-3:- Commercial arthmatic - Simple interest, compound interest inculding yearly and half yearly calculations, annuities, percentage, bill discount, ratio and proportation, duplicate, triplicate ratio and Sub duplicate ratio proportion - Third, fourth and inverse proportion problems.

Module -4:- Business Statistics - Meaning and importance of Statices measure of central tendency, Mean, median mode, gemotry mean, Harmon mean dispersion, range, Quartile deviat. mean deviation, Standard deviation and coefficient of Variation.

Module - 5 Business Statistics
Simple correlation and regression

Friday

Module - 1 Theory of equation

Importance of Business Mathematics:-

* It help in analysing financial performance of the business.
* It help in estimating the income and expenditure along with the risk analysis.

Equation :- Equality between two expression in x and y to form a equation.
Ex:- $2x + 3y = 0$

An equation is a relation between two or more variable hold good only for certain values of the variable.

Linear Equation :- The equation of degree one is called linear equation.
The Standard form of linear equation is $ax + by + c = 0$

Ex $2x + 7y = 0$
$4x + y = 0$

Quadratic Equation:- The equation of degree two such equation are called quadratic equation.
The standard form of quadratic equation $ax^2 + bx + c = 0$

Ex:- $3x^2 + 4x + 1 = 0$

1. Find x
i) $2x + 3 = 5$
$2x = 3 - 5$
$2x = 2$
$x = 2$ $\therefore x = 1$

ii) $2(x+3) - 6 = 17$

Sol) $2x + 6 - 6 = 17$

$2x = 17$

$x = 17/2$

iii) $7(x-3) - 3(x+4) = 7$

Sol) $7x - 21 - 3x + 12 = 7$

$4x = 7 + 21 + 12$

$4x = 40$

$x = 40/4$

$x = 20$

iv) $7(x-3) - 3(x+4) = 7 + 2(3x-8)$

$7x - 21 - 3x - 12 = 7 + 6x - 16$

$7x - 3x - 6x = 7 - 16 + 21 + 12$

$-2x = 24$

$x = 24/-2$

$x = -12$

v) $3(y-3) - (y+2) = 5(y-2) - 6(y-2)$

$3y - 9 - y + 2 = 5y - 10 - 6y + 12$

$3y - y - 5y + 6y = -10 + 12 + 9 + 2$

$3y = 13$

$y = 13/3$

vi) $(x+2)^2 - (x-3)^2 = 3(3x-3) - 15$

$= x^2 + 4 + 4x - [x^2 + 9 - 6x] = 3(3x-3) - 15$

$= x^2 + 4 + 4x - x^2 + 9 - 6x = 9x - 9 - 15$

$= 4 + 4x - 9 - 6x = 9x - 9 - 15$

$4x - 6x + 9x = 4 - 9 - 15 - 4 + 9$

$x = -19$

Formula

$(a+b)^2 = a^2 + b^2 + 2ab$

$(x+2)^2 = x^2 + 2^2 + 2 \times x \times 2$

$= x^2 + 4 + 4x$

$(a-b)^2 = a^2 + b^2 - 2ab$

$(x-3)^2 = x^2 + 3^2 - 2 \times x \times 3$

$x^2 + 9 - 6x$

vii) find a $\quad 2(a+3) = 10 + 4(a-8)$

$2a + 6 = 10 + 4a - 32$

$2a - 4a = 10 - 32 - 6$

$-2a = -28$

$a = 28/2$

$a = 14$

viii) $2(7+x) - 10 = 16 - 2(x-24)$

$14 + 2x - 10 = 16 - 2x + 48$

$2x + 2x = -14 + 10 - 16 + 48$

$2x + 2x = 16 + 48 - 14 + 10$

$4x = 60$

$x = 60/4$

$x = 15$

ix) Solve x

$[(x+2)+(x+3)]^2 - [(x+2)-(x+3)]^2 = 4x^2 + 64$

Sol) $[x+2+x+3]^2 - [x+2-x-3]^2 = 4x^2 + 64$

$(2x+5)^2 - (-1)^2 = 4x^2 + 64$

$4x^2 + 25 + 20x - 1 = 4x^2 + 64$

$20x = 64 - 25 + 1$

$20x = 40$

$x = 40/20$

$= 2$

$(a+b)^2 = a^2 + b^2 + 2ab$

$(2x+5)^2 = (2x)^2 + 5^2 + 2 \times 2x \times 5$

$4x^2 + 25 + 20x$

x) Solve for x

$2x + 3[5x - \{8 - (2x-1)\} + 6] = 3x - \{x + (5x-1)\} + 25$

$2x + 3[5x - \{8 - 2x + 1\} + 6] = 3x - \{x + 5x - 1\} + 25$

$2x + 3[5x - 8 + 2x - 1 + 6] = 3x - x - 5x + 1 + 25$

$2x + 15x - 24 + 6x - 3 + 18 = 3x - x - 5x + 1 + 25$

$2x + 15x + 6x - 3x + x + 5x = 1 + 25 + 24 + 3 - 18$

$26x = 35$

$x = 35/26$

xi) Solve for x

$3x - (3 + \{x - (3 - x)\}] = 5 + 2x$

$3x - [3 + \{x - 3 - x\}] = 5 + 2x$

$3x - [3 + x - 3 - x] = 5 + 2x$

$3x - 2x = 5$

$x = 5$

xii) Solve for x $\frac{3x-1}{2} + \frac{x+2}{3} = \frac{9x+12}{5}$

Sol

$\frac{9x - 3 + 2x + 4}{6} = \frac{9x + 12 \cdot 10}{5}$ $\quad \frac{9n - 3 + 2n + 4}{6} = \frac{9n + 12}{5}$

$\frac{4x+1}{6} \times \frac{9x+2}{5}$ $\quad \frac{11n+1}{6} = \frac{9n+2}{5}$

$5(11n+1) = 6(9n+2)$

$5(4x+1) = 6(9x+2)$ $\quad 55n + 5 = 54n + 12$

$55x + 5 = 54x + 12$ $\quad 55n - 54n = 12 - 5$

$55x - 54x = 12 - 5$ $\quad n = 7$

$x = 7/1$

xiii) Solve $\frac{4x+5}{6x-1} \times \frac{3+2x}{3x-4}$

Sol $(4x+5)(3x-4) = (6x-1)(3+2x)$

$12x^2 - 16x + 15x - 20 = 18x + 12x^2 - 3 - 2x$

$-16x + 15x - 18x + 2x = -3 + 20$

$= 17x = 17$

$x = 17/17$

$\therefore x = 1$

Simulataneous equation:-

Types of simulataneous equation:-

- Imp ✓ Elimination method — 8 marks
- Imp ✓ Substitution method
- Cross multiplication method

I 1) Method Elimination:-

1) Solve by elimination method

$2x - 3y = 19$

$3x + 2y = 9$

Sol $2x - 3y = 19 \rightarrow (1) \times 3$

$3x + 2y = 9 \rightarrow (2) \times 2$

$6x - 9y = 57 \rightarrow (3)$

$6x + 4y = 18 \rightarrow (4)$

$-13y = 39$

$y = \frac{39}{-13}$

$y = -3$

Sub $y = -3$ in (1)

$2x - 3y = 19$

$2x - 3(-3) = 19$

$2x + 9 = 9$

$2x = 19 - 9$

$2x = 10$

$x = 10/2$

$x = 5$

Varification:-

Sub Eq 2 with $x = 5$

$3x + 2y = 9$

$3(5) + 2(-3) = 9$

$15 - 6 = 9$

$9 = 9$

$\therefore LHS = RHS$

) $2x + 3y = 42 \rightarrow ① \times 5$
$5x - y = 20 \rightarrow ② \times 2$

$10x + 15y = 210 \rightarrow ③$
$10x - 2y = 40 \rightarrow ④$
$17y = 170$

$y = \frac{170}{17}$

$\boxed{y = 10}$

Sub equ 1 in $y = 10$

$2x + 3(10) = 42$

$2x + 30 = 42$

$2x = 42 - 30$

$2x = 12$

$x = 12/2$

$\boxed{x = 6}$

Eq ②

$5(6) - 10 = 20$
$30 - 10 = 20$
$20 = 20$
$\therefore$ LHS = RHS

3) $3x - 4y = 8 \rightarrow ① \times 1$
$x + 3y = 4 \rightarrow ② \times 3$

Sol) $3x - 4y = 8$
$3x + 9y = 12$

$3x - 4y = 8$
$3x + 9y = 12$
$-13y = -4$

$y = \frac{-4}{-13}$

$y = 0.30$

Sub $y = 0.30$ in eq ①

$3x - 4y = 8$

$3x - 4(0.30) = 8$
$3x - 1.2 = 8$
$3x = 8 + 1.2$

$x = \frac{9.2}{3}$

$x = 3.06$

Sub $x = 3.06$ eq ②

$3.06 + 3(0.30) = 4$
$3.06 + 0.9 = 4$

$y = 4/13$

Sub $y = 4/13$ in eq ①
$3x - 4y = 8$
$3x - 4(4/13) = 8$
$3x - \frac{16}{13} = 8$

$3x = 8 + \frac{16}{13}$

$3x = \frac{104 + 16}{13} = \frac{120}{13}$

$x = \frac{\cancel{120}^{40}}{13 \times \cancel{3}}$

$x = 40/13$

4) Solve by elimination method

sol: $3x + 7y = 13 \rightarrow (1)$
$5x - 2y = 8 \rightarrow (2)$

$3x + 7y = 13 \rightarrow \times 5$
$5x - 2y = 8 \rightarrow \times 3$

$15x + 35y = 65$
$(-)\ 15x \ \underset{+}{-}\ 6y = \underset{-}{24}$

$41y = 41$

$y = 41/41$

$y = 1$

Sub eq (1) in 1

$3x + 7(1) = 13$
$3x + 7 = 13$
$3x = 13 - 7$
$3x = 6$
$x = 6/3$
$x = 2$

Verification :-
Sub eq 2 in x 2

$5x - 2y = 8$
$5(2) - 2(1) = 8$
$10 - 2 = 8$
$8 = 8$

5) Solve by elimination method

sol: $8x - 3y = 3 \rightarrow (1)$
$6x - 6y = 1 \rightarrow (2)$

$8x - 3y = 3 \rightarrow 6$
$6x - 6y = 1 \rightarrow 8$

$48x - 18y = 18$
$- 48x \ \underset{+}{-}\ 48y = \underset{-}{8}$

$30y = 10$

$30y = 10$

$y = \frac{10}{30}$

$y = \frac{10}{30}$ $y = 1/3$

Sub eq (1) in y $1/3$

$8x - 3y = 3 \rightarrow (1)$
$8x - 3(3) = 3$
$8x - 9 = 3$
$8x = 9 + 3$
$8x = 12$
$x = 12/8$

$8x - 3(10/30) = 3$
$8x - \frac{30}{30} = 3$
$8x = \frac{3 + 30}{30}$

$8x - \frac{3}{3} = 3$
$8x - 1 = 3$
$8x = 3 + 1$
$8x = 4$
$x = \frac{4}{8}$
$x = 1/2$

II Method of substitution:-

1. Solve by substitution method

Sol: $x+2y=7$
$2x-y=4$

Sol: $x+2y=7 \rightarrow ①$
$2x-y=4 \rightarrow ②$

Eq ① $x+2y=7$
$x=7-2y \rightarrow ③$

Sub $x=7-2y$ in eq ②

$2x-y=4$
$2(7-2y)-y=4$
$14-4y-y=4$
$14-5y=4$
$-5y=4-14$
$\cancel{-}5y=\cancel{-}10$
$5y=10$
$y=\frac{10}{5}$
$y=2$

Sub $y=2$ in eq 3
$x=7-2y$
$x=7-2(2)$
$x=7-4$
$x=3$

2. Solve by Substitution method

$4x-y=2$
$-3x+2y=1$

Sol, $4x-y=2 \longrightarrow ①$
$-3x+2y=1 \longrightarrow ②$

Eq ① $4x-y=2$
$4x=2+y$
$x=\frac{2+y}{4} \rightarrow ③$

Sub $x=\frac{2+y}{4}$ in eq ②

$-3x+2y=1$

$-3\left(\frac{2+y}{4}\right)+2y=1$

$\frac{-6-3y}{4}+\frac{2y}{1}=1$

$\frac{-6-3y+8y}{4} \times \frac{1}{1}$

$-6-3y+8y=4$
$-6+5y=4$
$5y=4+6$
$5y=10$
$y=\frac{10}{5}$

$y=2$

Sub $y=2$ in eq ③

$x=\frac{2+y}{4}$

$x=\frac{2+2}{4}=\frac{4}{4}$

$x=1$

3. Solve by Substitution method

$3x + 7y = 13 \rightarrow (1)$
$5x - 2y = 8 \rightarrow (2)$

Sol, eq $3x + 7y = 13$
$3x = 13 - 7y$
$x = \frac{13-7y}{3} \rightarrow (3)$

Substitution $x = \frac{13-7y}{3}$ in eq (2)

$5\left(\frac{13-7y}{3}\right) - 2y = 8$

$\frac{65-35y}{3} - 2y = 8$

$\frac{65-35y-6y}{3} = \frac{8}{1}$

$65 - 35y - 6y = 24$
$65 - 41y = 24$
$-41y = 24 - 65$
$-41y = -41$
$y = \frac{-41}{-41} = 1$

$y = 1$

sub $y=1$ in eq (3) $x = \frac{13-7y}{3} = \frac{13-7(1)}{3} = \frac{13-7}{3}$

$\frac{6}{3} = 2$

III Cross multiplication method : Consider two equation

$a_1x + b_1y + c_1 = 0 \rightarrow (1)$
$a_2x + b_2y + c_2 = 0 \rightarrow (2)$

$\frac{x}{b_1c_2 - b_2c_1} = \frac{y}{c_1a_2 - c_2a_1} = \frac{1}{a_1b_2 - a_2b_1}$

$\Rightarrow x = \frac{b_1c_2 - b_2c_1}{a_1b_2 - a_2b_1}, \; y = \frac{c_1a_2 - c_2a_1}{a_1b_2 - a_2b_1}$

1. Solve by method of cross multiplication

$x + 2y - 4 = 0$
$3x + y - 7 = 0$

formulae :

$x = \frac{b_1c_2 - b_2c_1}{a_1b_2 - a_2b_1}$ $y = \frac{c_1a_2 - c_2a_1}{a_1b_2 - a_2b_1}$

Give $a_1 = 1$
$b_1 = 2$
$c_1 = -4$
$a_2 = 3$
$b_2 = 1$
$c_2 = -7$

$x = \frac{b_1c_2 - b_2c_1}{a_1b_2 - a_2b_1}$

$= \frac{(2\times-7) - (1\times-4)}{(1\times1) - (3\times2)}$

$= \frac{-14+4}{1-6}$

$= \frac{-10}{-5} \quad x = 2$

$y = \frac{c_1a_2 - c_2a_1}{a_1b_2 - a_2b_1}$

$= \frac{(-4\times3) - (-7\times1)}{(1\times1) - (3\times2)}$

$= \frac{-12+7}{1-6}$

$= \frac{-5}{-5}$ $y = 1$

2. Solve by cross multiplication method

$10x - 9y = 12 \Rightarrow 10x - 9y - 12 = 0$
$3x - 9y = 17 \qquad 3x - 9y - 17 = 0$

Given
$a_1 = 10$
$b_1 = -9$
$c_1 = -12$
$a_2 = 3$
$b_2 = -9$
$c_2 = -17$

$x = \frac{b_1c_2 - b_2c_1}{a_1b_2 - a_2b_1}$

$= \frac{(-9\times-17) - (-9\times-12)}{(10\times-9) - (3\times-9)}$

$x = \frac{153 - 108}{-90 + 27}$

$x = \frac{45}{-63} = \frac{-5}{7}$

$y = \frac{c_1a_2 - c_2a_1}{a_1b_2 - a_2b_1}$

$y = \frac{(-12\times3) - (-17\times10)}{(10\times-9) - (3\times-9)}$

$= \frac{-36 + 170}{-90 + 27}$

$= \frac{134}{-63} = \frac{-134}{63}$

3. Solve by Cross multiplication method

$2x - 3y = 19 \Rightarrow 2x - 3y - 19 = 0$
$3x + 2y = 9 \qquad 3x + 2y - 9 = 0$

Given,
$a_1 = 2$
$b_1 = -3$
$c_1 = -19$
$a_2 = 3$
$b_2 = 2$
$c_2 = -9$

$x = \frac{b_1c_2 - b_2c_1}{a_1b_2 - a_2b_1}$

$= \frac{(-3\times-9) - (2\times-19)}{(2\times2) - (3\times-3)}$

$x = \frac{27 + 38}{4 + 9}$

$x = \frac{65}{13}$

$x = 5$

$y = \frac{c_1a_2 - c_2a_1}{a_1b_2 - a_2b_1}$

$= \frac{(-19\times3) - (-9\times2)}{(2\times2) - (-3\times3)}$

$y = \frac{-57 + 18}{4 + 9}$

$y = \frac{-39}{13}$

$y = -3$

Quadratic equation: The equation of the form $ax^2+bx+c=0$ containing the degree two is called quadratic equation. The general form quadratic equation is $ax^2+bx+c=0$

Types of Quadratic equation: They are (solving the method)

* Pure quadratic equation
* Adfected quadratic equation

I 1) Pure quadratic equation: So
$3x^2 - 16x + 5 = 0$

Sol) $3x^2 - 16x + 5 = 0$

1st method:

$3x^2 - 1x - 15x + 5 = 0$

$x(3x-1) - 5(3x-1) = 0$

$(3x-1)(x-5) = 0$

$3x - 1 = 0 \mid x - 5 = 0$

$x = 1/3 \mid x = 5$

2nd method:

$$\begin{pmatrix} 3x & & -1 \\ & \times & \\ x & & -5 \end{pmatrix} \qquad \begin{array}{r} -15x \\ -1x \\ \hline -16x \end{array}$$

$(3x-1) \mid (x-5)$

$3x - 1 = 0 \mid x - 5 = 0$

$3x = 1 \mid x = 5$

$x = 1/3$

2 Solve $4x^2 = 11x + 3$

Sol) $4x^2 - 11x - 3 = 0$

$$4x \begin{pmatrix} 4x & & +1 \\ & \times & \\ x & & -3 \end{pmatrix} \qquad \begin{array}{r} -12x \\ +1x \\ \hline -11x \end{array}$$

$(4x+1) \mid (x-3)$

$4x + 1 = 0 \mid x - 3 = 0$

$x = -1/4 \mid x = 3$

3) Solve, $x^2 + 6x + 8 = 0$

Sol)

$$\begin{pmatrix} x & & 4 \\ & \times & \\ x & & 2 \end{pmatrix} \qquad \begin{array}{r} +2x \\ +4x \\ \hline +6x \end{array}$$

$(x+4) \mid (x+2)$

$(x+4) = 0 \mid (x+2) = 0$

4) Solve $2x^2 - 7x + 3 = 0$

Sol)

$$\begin{pmatrix} 2x & & -1 \\ & \times & \\ x & & -3 \end{pmatrix} \qquad \begin{array}{r} -6x \\ -1x \\ \hline -7x \end{array}$$

$(2x-1)(x-3) = 0$

$2x - 1 = 0 \mid x - 3 = 0$

$x = 1/2 \mid x = 3$

5. Solve $(5x-3)(5x+3) = 16$

Sol) $25x^2 - 9 = 16$

$25x^2 = 16 + 9$

$25x^2 = 25$

$x^2 = 25/25$

$x^2 = 1$

Take square root both the sides

$\sqrt{x^2} = \pm\sqrt{1}$

$x = \pm 1$

formulae:

$(a-b)(a+b) = a^2 - b^2$

$= (5x)^2 - (3)^2$

$= 25x^2 - 9$

II Adfected Quadratic equation: An adfected quadratic equation of the form $ax^2 + bx + c = 0$ can be solved by

* factorization method
* Imp ✓ Method of completing of square
* Imp ✓ Formulae method

1) Method of completing of square:

Solve, $4x^2 + 4x - 3 = 0$

Sol. $4x^2 + 4x - 3 = 0$ $\quad (a+b)^2 = a^2 + b^2 + 2ab$

$\div$ the whole equation by 4 $\quad (x+\frac{1}{2})^2 = x^2 + (\frac{1}{2})^2 + 2 \cdot x \cdot \frac{1}{2}$

$= x^2 + (\frac{1}{2})^2 + x$

$1 \times \frac{1}{2} = \frac{1}{2}$

$x^2 + x - \frac{3}{4} = 0$

Add $(\frac{1}{2})^2$ on both side

$x^2 + x + (\frac{1}{2})^2 - \frac{3}{4} = (\frac{1}{2})^2$

$(x+\frac{1}{2})^2 = (\frac{1}{2})^2 + \frac{3}{4}$

$(x+\frac{1}{2})^2 = \frac{1}{4} + \frac{3}{4} = \frac{4}{4}$

$(x+\frac{1}{2})^2 = 1$

Taking square root both the side

$\sqrt{(x+\frac{1}{2})^2} = \pm\sqrt{1}$

$x + \frac{1}{2} = \pm 1$

$x + \frac{1}{2} = +1$	$x + \frac{1}{2} = -1$
$x = 1 - \frac{1}{2}$	$x = -1 - \frac{1}{2}$
$x = \frac{1}{2}$	$= \frac{-2-1}{2}$
	$x = -\frac{3}{2}$

2. Solve by method of completing square
$3x^2 - 5x - 8 = 0$

Sol. $\div$ Whole equation by 3

Sol. $x^2 - \frac{5}{3}x - \frac{8}{3} = 0$ $\quad \boxed{\frac{5}{3} \times \frac{1}{2} = \frac{5}{6}}$

$(a-b)^2 = a^2 + b^2 - 2ab$

$(x - \frac{5}{6})^2 = x^2 + (\frac{5}{6})^2 - 2 \times x \times \frac{5}{6}$

$= x^2 + (\frac{5}{6})^2 - \frac{5}{3}x$

Adding $(\frac{5}{6})^2$ on both side

$x^2 - \frac{5}{3}x + (\frac{5}{6})^2 - \frac{8}{3} = (\frac{5}{6})^2$

$(x - \frac{5}{6})^2 = (\frac{5}{6})^2 + \frac{8}{3}$

$= \frac{25}{36} + \frac{8}{3}$ LCM

LCM: $3 \mid 3, 36$; $3 \mid 1, 12$; $4 \mid 1, 4$; $1, 1$ $= 36$

$\frac{25}{36} \times 36 \quad \frac{8}{3} \times 36^{12}$

$= \frac{25 + 96}{36} = \frac{121}{36}$

$(x - \frac{5}{6})^2 = \frac{121}{36}$

Take square roots

$\sqrt{(x - \frac{5}{6})^2} = \pm\sqrt{\frac{121}{36}}$

$x - \frac{5}{6} = \pm\frac{11}{6}$

$x - \frac{5}{6} = +\frac{11}{6}$	$x - \frac{5}{6} = \frac{-11}{6}$
$x - \frac{5}{6} = +\frac{11}{6}$	$= \frac{-11}{6} + \frac{5}{6}$
$x = \frac{11}{6} + \frac{5}{6}$	$= -\frac{6}{6}$
$= \frac{16}{6} = \frac{8}{3}$	$= -1$

3. Solve by the method of completing square
$x^2+9x+7=0$

Sol) Add $9/2$ both the side $\qquad 9x = 9 \times 1/2 = 9/2$

$$x^2+9x+(9/2)^2+7=(9/2)^2$$

$$(a+b)^2 = a^2+b^2+2ab$$
$$(x+9/2)^2 = x^2+9/2^2+2\times x \times 9/2$$
$$(x+9/2)^2 = x^2+(9/2)^2+9x$$

$$(x+9/2)^2 = (9/2)^2 - 7$$

$$= \frac{81}{4} - 7$$

$$= \frac{81-28}{4}$$

$$= \frac{53}{4}$$

$$(x+9/2)^2 = \frac{53}{4}$$

Taking square root both side

$$\sqrt{(x+9/2)^2} = \pm\sqrt{53/4}$$

$$x+9/2 = \pm\sqrt{53/2}$$

$x+9/2 = +\sqrt{53/2}$	$x+9/2 = -\sqrt{53/2}$
$x = \sqrt{53/2} - 9/2$	$x = -\sqrt{53/2} - 9/2$
$= \frac{\sqrt{53}-9}{2}$	$= \frac{\sqrt{53}-9}{2}$

4) $x^2-5x-7=0$

Add $5/2$ both the sides $\qquad 5\times 1/2 = 5/2$

$x^2 - 5/2 \ldots$

$$x^2-5x-(5/2)^2-7=(5/2)^2$$

$$(a-b)^2 = a^2+b^2-2ab$$
$$(x-5/2)^2 = x^2(5/2)^2-2x\,x$$
$$(x-5/2)^2 = x^2+(5/2)^2-5x$$

$$(x-5/2)^2 = (5/2)^2+7$$

$$= \frac{25}{4}+7$$

$$= \frac{25+28}{4}$$

$$= \frac{53}{4}\ \frac{53}{4}$$

$$(x-5/2)^2 = \frac{53}{4}$$

Taking square root both side

$$\sqrt{(x-5/2)^2} = \pm\sqrt{\frac{53}{4}}$$

$$x-5/2 = \pm\sqrt{53/2}$$

$x+5/2 = +\sqrt{53/2}$	$x+5/2 = -\sqrt{53/2}$
$x = \sqrt{53/2} - 5/2$	$x = \mp\sqrt{53/2} + 5/2$
$= \frac{+\sqrt{53}+5}{2}$	$= \frac{-\sqrt{53}+5}{2}$

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II Formulae method:

The root of equation can be obtained by using the formulae $x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

The general form of equation is $ax^2+bx+c=0$

1. Solve by formulae method
$9x^2-3x-2=0$
The equ. of the form $ax^2+bx+c=0$
$a=9$
$b=-3$
$c=-2$

$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$= \frac{-(-3) \pm \sqrt{(-3)^2-4\times 9\times(-2)}}{2(9)}$

$x = \frac{+3 \pm \sqrt{9+72}}{18}$

$x = \frac{3 \pm \sqrt{81}}{18}$

$x = \frac{3 \pm 9}{18}$

$x = \frac{3+9}{18}$	$x = \frac{3-9}{18}$
$= \frac{\cancel{12}\,2}{\cancel{18}\,3}$	$= \frac{-\cancel{6}\,1}{\cancel{18}\,3}$
$= 2/3$	$= -1/3$

2. $x^2+x-6=0$
The equ. of the form $ax^2+bx+c=0$
$a=1$
$b=1$
$c=-6$

$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$= \frac{-1 \pm \sqrt{(1)^2-4(1)\times(-6)}}{2}$

$x = \frac{-1 \pm \sqrt{1+24}}{2}$

$x = \frac{-1 \pm \sqrt{25}}{2}$

$x = \frac{-1 \pm \sqrt{5}}{2}$

$x = \frac{-1+5}{2}$	$x = \frac{-1-5}{2}$
$x = 4/2$	$x = -6/2$, $x = \frac{\cancel{-6}\,3}{\cancel{2}}$
$= 2$	$= -3$

3. $12x^2-23x-24=0$
Sol. The equ. of the form $ax^2-bx+c=0$
$a=12$
$b=-23$
$c=-24$

$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$= \frac{-(-23) \pm \sqrt{(-23)^2-4\times 12\times(-24)}}{2(12)}$

$x = \frac{+23 \pm \sqrt{529+1152}}{24}$	$x = \frac{-23 \pm \sqrt{1,681}}{24}$

$x = \frac{23 \pm \sqrt{41}}{24}$

$x = \frac{23-41}{24}$	$\frac{23+41}{24}$
$= \frac{-18}{24}$	$= \frac{64}{24}$
$= -\frac{3}{4}$	$= \frac{8}{3}$

4. $5(x-2)^2 - 6 = -13(x-2)$

Sol: $5(x^2+4-4x) - 6 = -13x + 26$

$5x^2 + 20 - 20x - 6 + 13x - 26 = 0$

$5x^2 - 7x - 12 = 0$

$(a-b)^2 = a^2 + \ldots$

$(x-2)^2 = x^2 + 2^2 - \ldots$

$a = 5$

$b = -7$

$c = -12$ Th square of the form of $ax^2 + bx + c = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-7 \pm \sqrt{(-7)^2 - 4 \times 5 \times (-12)}}{2(5)}$$

$$x = \frac{7 \pm \sqrt{49 + 240}}{10}$$

$$= \frac{7 \pm \sqrt{289}}{10}$$

$$= \frac{7 \pm 17}{10}$$

$\frac{7+17}{10}$	$\frac{7-17}{10}$
$= \frac{24}{10}$	$= \frac{-10}{10}$
$x = \frac{12}{5}$	$x = -1$

Module-2 Indian Matrices and togan Determinants

Matrix:- A rectangular arrangement of numbers in the form of rows and columns (row) horizontal (column) vertical

Ex:- $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$

Order of matrix:- The number of row and no. of columns is called order of matrix.

Ex: $\begin{bmatrix} 2 & 4 & 5 \\ 6 & 7 & 8 \end{bmatrix}_{2\times 3}$

Types of matrices

* **Row matrices :-** If matrix as only one row then it is called row matrix Ex:- $[3\ 4\ 2]_{1\times 3}$

* **Column matrices:-** If matrix as only one column then it is called column matrices Ex:- $\begin{bmatrix} 5 \\ 6 \\ 8 \end{bmatrix}_{3\times 1}$

* **Square matrices:-** It matrices in which number of row and number columns are equal is called square matrix

  Ex: $\begin{bmatrix} 1 & 2 \\ 6 & 7 \end{bmatrix}_{2\times 2}$

* **Diagonal matrices:-** If matrix square matrise having non-zero having elements in principle diagonal and remaining all the elements are equal to zero

  Ex: $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 5 & 8 \\ 0 & 0 & 8 \end{bmatrix}$, $B = \begin{bmatrix} 5 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

* **Unit / Identity matrices (I):** A square matrix is said to be identity matrices if all principle diagonal elements are one.

  Ex $I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}_{3\times 3}$ $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}_{2\times 2}$

* **Scalar matrices :-** A diagonal matrise is said to be scalar if all principle diagonal elements is same

  $B = \begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix}_{3\times 3}$ $A \begin{bmatrix} \sqrt{3} & 0 & 0 \\ 0 & \sqrt{3} & 0 \\ 0 & 0 & \sqrt{3} \end{bmatrix}$

* **Zero matrices or null matrices or void matrix :-** In matrices in which all the elements are zero is called zero matrices or null matrix or void matrix

  Ex: $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}_{3\times 3}$ $B = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}_{2\times 2}$

* **Upper triangular matrix :-** Square matrix said to be a upper triangular matrix if all the elements below the principle diagonal elements are zero.

  $A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 5 & 4 \\ 0 & 0 & 9 \end{bmatrix}$

* **Lower triangular matrix :-** Square matrix said to be a lower triangular matrix if all the elements above the principle diagonal are zero

  $B \begin{bmatrix} 1 & 0 & 0 \\ 2 & 5 & 0 \\ 2 & 1 & 8 \end{bmatrix}$

* Transpose of matrix:- A matrix obtained by interchanging rows and columns the called transpose of matrix. its denoted by A' or A^T

Ex. I If $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$

the A' or $A' = \begin{bmatrix} 1 & 4 & 7 \\ 2 & 5 & 8 \\ 3 & 6 & 9 \end{bmatrix}$

* II If $B = \begin{bmatrix} 3 & 2 & 4 \\ 1 & 4 & 3 \end{bmatrix}_{2\times3}$ $B' = \begin{bmatrix} 3 & 1 \\ 2 & 4 \\ 4 & 3 \end{bmatrix}_{3\times2}$

* Symmetric matrices:- Any square matrix is said to be symmetric matrix if it satisfied the condition $A = A'$

Ex: $A = \begin{bmatrix} 1 & 5 & 3 \\ 5 & 2 & 6 \\ 3 & 6 & 4 \end{bmatrix}$ $A' = \begin{bmatrix} 1 & 5 & 3 \\ 5 & 2 & 6 \\ 3 & 6 & 4 \end{bmatrix}$

* Equality matrices;- Two matrices of the same order are said to be equal only when the corresponding element are equal.

Ex If $A = \begin{bmatrix} 3 & -4 & 2 \\ -1 & 4 & 7 \end{bmatrix}$

$B = \begin{bmatrix} 3 & -1 \\ -4 & 4 \end{bmatrix}$

$C = \begin{bmatrix} 3 & -4 & 2 \\ -1 & 4 & 7 \end{bmatrix}$

$A = C$

I Addition and Subtraction of Matrices:-

1 If $A = \begin{bmatrix} 5 & 3 \\ 4 & 6 \end{bmatrix}$ and $B \begin{bmatrix} 6 & 8 \\ 9 & 1 \end{bmatrix}$ $(A+B)$ and $(A-B)$

$2A + 3B$

$A+B = \begin{bmatrix} 5 & 3 \\ 4 & 6 \end{bmatrix} + \begin{bmatrix} 6 & 8 \\ 9 & 1 \end{bmatrix}$

$A+B = \begin{bmatrix} 11 & 11 \\ 13 & 7 \end{bmatrix}$

$A-B \begin{bmatrix} 5 & 3 \\ 4 & 6 \end{bmatrix} - \begin{bmatrix} 6 & 8 \\ 9 & 1 \end{bmatrix}$

$A-B = \begin{bmatrix} -1 & -5 \\ -5 & 5 \end{bmatrix}$

$2A + 3B = 2\begin{bmatrix} 5 & 3 \\ 4 & 6 \end{bmatrix} + 3\begin{bmatrix} 6 & 8 \\ 9 & 1 \end{bmatrix}$

$= \begin{bmatrix} 10 & 6 \\ 8 & 12 \end{bmatrix} + \begin{bmatrix} 18 & 24 \\ 27 & 3 \end{bmatrix}$

$= \begin{bmatrix} 28 & 30 \\ 35 & 15 \end{bmatrix}$

2 If $A = \begin{bmatrix} 0 & 1 & 2 \\ 2 & 1 & 4 \end{bmatrix}$ $B \begin{bmatrix} 7 & 6 & 3 \\ 1 & 4 & 5 \end{bmatrix}$ find $A+B$, $5A - 3A$

$6A + 2B$

$A+B = A \begin{bmatrix} 0 & 1 & 2 \\ 2 & 1 & 4 \end{bmatrix} + B \begin{bmatrix} 7 & 6 & 3 \\ 1 & 4 & 5 \end{bmatrix}$

$A+B = \begin{bmatrix} 7 & 7 & 5 \\ 3 & 5 & 9 \end{bmatrix}$

5A - 3B

$$5\begin{bmatrix}0 & 1 & 2\\2 & 1 & 4\end{bmatrix} - 3\begin{bmatrix}7 & 6 & 3\\1 & 4 & 5\end{bmatrix}$$

$$\begin{bmatrix}0 & 5 & 10\\10 & 5 & 20\end{bmatrix} - \begin{bmatrix}21 & 18 & 9\\3 & 12 & 15\end{bmatrix}$$

$$5A - 3B = \begin{bmatrix}-21 & -13 & 1\\7 & -7 & 5\end{bmatrix}$$

6A + 2B

$$6\begin{bmatrix}0 & 1 & 2\\2 & 1 & 4\end{bmatrix} + 2\begin{bmatrix}7 & 6 & 3\\1 & 4 & 5\end{bmatrix}$$

$$\begin{bmatrix}0 & 6 & 12\\12 & 6 & 24\end{bmatrix} + \begin{bmatrix}14 & 12 & 6\\2 & 8 & 10\end{bmatrix}$$

$$6A + 2B = \begin{bmatrix}14 & 18 & 18\\14 & 14 & 34\end{bmatrix}$$

3. If $A = \begin{bmatrix}2 & 0 & 4\\6 & 2 & 8\\2 & 4 & 6\end{bmatrix}$, $B = \begin{bmatrix}8 & 4 & 2\\0 & 2 & 1\\2 & 2 & 5\end{bmatrix}$, $C = \begin{bmatrix}8 & 0 & 2\\1 & 2 & -6\\-8 & 4 & -3\end{bmatrix}$

find 1) $A+B$, 2) $A+C$ 3) $A-B$ 4) $A'+C'$ 5) $(A+B)'$ 6) $B'+2C$ 7) $A+2B-2C$

1) $A+B$

$$A\begin{bmatrix}2 & 0 & 4\\6 & 2 & 8\\2 & 4 & 6\end{bmatrix} + B\begin{bmatrix}8 & 4 & 2\\0 & 2 & 1\\2 & 2 & 5\end{bmatrix}$$

$$A+B = \begin{bmatrix}10 & 4 & 6\\6 & 4 & 9\\4 & 6 & 11\end{bmatrix}$$

$A+C$:

$$A\begin{bmatrix}2 & 0 & 4\\6 & 2 & 8\\2 & 4 & 6\end{bmatrix} + C\begin{bmatrix}8 & 0 & 2\\1 & 2 & -6\\-8 & 4 & -3\end{bmatrix}$$

$$A+C = \begin{bmatrix}10 & 0 & 6\\7 & 4 & -14\\-10 & 8 & -9\end{bmatrix}$$

$$A-B = A\begin{bmatrix}2 & 0 & 4\\6 & 2 & 8\\2 & 4 & 6\end{bmatrix} - B\begin{bmatrix}8 & 4 & 2\\0 & 2 & 1\\2 & 2 & 5\end{bmatrix}$$

$$A-B = \begin{bmatrix}-6 & -4 & 2\\6 & 0 & 7\\0 & -2 & 1\end{bmatrix}$$

$A'+C'$:

$$A'\begin{bmatrix}2 & 0 & 4\\6 & 2 & 8\\2 & 4 & 6\end{bmatrix} + C'\begin{bmatrix}8 & 0 & 2\\1 & 2 & -6\\-8 & 4 & -3\end{bmatrix}$$

$$A'\begin{bmatrix}2 & 6 & 2\\0 & 2 & 4\\4 & 8 & 6\end{bmatrix} + C'\begin{bmatrix}8 & 1 & -8\\0 & 2 & 4\\2 & -6 & -3\end{bmatrix}$$

$$A'+C' = \begin{bmatrix}10 & 7 & -10\\0 & 4 & 8\\ \text{[illegible]} & \text{[illegible]} & \text{[illegible]}\end{bmatrix}$$

$A' + B' = A + B \begin{bmatrix} 10 & 4 & 6 \\ 6 & 4 & 9 \\ 4 & 6 & 11 \end{bmatrix}$

$(A+B)' = \begin{bmatrix} 10 & 6 & 4 \\ 4 & 4 & 6 \\ 6 & 9 & 11 \end{bmatrix}$

$B' + 2C = \begin{bmatrix} 8 & 0 & 2 \\ 4 & 2 & 2 \\ 2 & 1 & 5 \end{bmatrix} + \begin{bmatrix} 16 & 0 & 4 \\ 2 & 4 & 12 \\ 16 & 8 & 6 \end{bmatrix}$

$B' + 2C = \begin{bmatrix} 24 & 0 & 6 \\ 6 & 6 & 10 \\ 14 & 9 & 1 \end{bmatrix}$

$A + 2B - 2C = \begin{bmatrix} 2 & 0 & 4 \\ 6 & 2 & 8 \\ 2 & 4 & 6 \end{bmatrix} + \begin{bmatrix} 16 & 8 & 4 \\ 0 & 4 & 2 \\ 4 & 4 & 10 \end{bmatrix} - \begin{bmatrix} 16 & 0 & 4 \\ 2 & 4 & 12 \\ 16 & 8 & 6 \end{bmatrix}$

$= \begin{bmatrix} 2 & 8 & 4 \\ 4 & 2 & 2 \\ 22 & 0 & 22 \end{bmatrix}$

4. If matrix $A = \begin{bmatrix} 9 & 1 \\ 4 & 3 \end{bmatrix}$ $B = \begin{bmatrix} 1 & 5 \\ 7 & 12 \end{bmatrix}$ find matrix X such that, $2A + 5B + 2X = 0$

Sol) Let $2A + 5B + 2X = 0$

$2X = -2A - 5B$

$= -2\begin{bmatrix} 9 & 1 \\ 4 & 3 \end{bmatrix} - 5\begin{bmatrix} 1 & 5 \\ 7 & 12 \end{bmatrix}$

$2X = \begin{bmatrix} -18 & -2 \\ -8 & -6 \end{bmatrix} + \begin{bmatrix} -5 & -25 \\ -35 & -60 \end{bmatrix}$

$2X = \begin{bmatrix} -23 & -27 \\ -43 & -66 \end{bmatrix}$

$X = \dfrac{\begin{bmatrix} -23 & -27 \\ -43 & -66 \end{bmatrix}}{2}$

$X = \begin{bmatrix} -23/2 & -27/2 \\ -43/2 & -66/2 \end{bmatrix} = \begin{bmatrix} -23/2 & -27/2 \\ -43/2 & -33 \end{bmatrix}$

5. Solve A and B if $A + 2B = \begin{bmatrix} 21 & 16 \\ 21 & 2 \end{bmatrix}$ and $2A + 3B = \begin{bmatrix} -12 & -11 \\ 1 & -16 \end{bmatrix}$

$A + 2B = \begin{bmatrix} 21 & 16 \\ 21 & 2 \end{bmatrix} \to ① \times 2$, $2A + 3B = \begin{bmatrix} -21 & -11 \\ 1 & -16 \end{bmatrix} \to ② \times 1$

$2A + 4B = \begin{bmatrix} 42 & 32 \\ 42 & 4 \end{bmatrix}$

$2A + 3B = \begin{bmatrix} -12 & -11 \\ 1 & -16 \end{bmatrix}$

$(-) \quad (-) \quad (-)$

$B = \begin{bmatrix} 42 & 32 \\ 42 & 4 \end{bmatrix} - \begin{bmatrix} -12 & -11 \\ 1 & -16 \end{bmatrix}$

$$B = \begin{bmatrix} -54 & 43 \\ 41 & 20 \end{bmatrix}$$

Eq ① →

$$A + 2B = \begin{bmatrix} 21 & 16 \\ 21 & 2 \end{bmatrix}$$

$$A = \begin{bmatrix} 21 & 16 \\ 21 & 2 \end{bmatrix} - 2B$$

$$A = \begin{bmatrix} 21 & 16 \\ 21 & 2 \end{bmatrix} - 2\begin{bmatrix} 54 & 43 \\ 41 & 20 \end{bmatrix}$$

$$A = \begin{bmatrix} 21 & 16 \\ 21 & 2 \end{bmatrix} - \begin{bmatrix} 108 & 86 \\ 82 & 40 \end{bmatrix}$$

$$A = \begin{bmatrix} -87 & -70 \\ -61 & -38 \end{bmatrix}$$

II. Multiplication of matrices

1. Solve AB if $A = \begin{bmatrix} 4 & 2 \\ 6 & 4 \end{bmatrix}$, $B = \begin{bmatrix} 8 & 4 \\ 6 & 2 \end{bmatrix}$

Sol: $AB = \begin{bmatrix} 4 & 2 \\ 6 & 4 \end{bmatrix}\begin{bmatrix} 8 & 4 \\ 6 & 2 \end{bmatrix}$

$$= \begin{bmatrix} 4\times8 + 2\times6 & 4\times4 + 2\times2 \\ 6\times8 + 4\times6 & 6\times4 + 4\times2 \end{bmatrix}$$

$$= \begin{bmatrix} 32 + 12 & 16 + 4 \\ 48 + 24 & 24 + 8 \end{bmatrix}$$

$$AB = \begin{bmatrix} 44 & 20 \\ 72 & 32 \end{bmatrix}$$

2. If $A = \begin{bmatrix} 3 & 1 \\ 6 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 0 \\ 1 & 3 \end{bmatrix}$ find AB and BA

Sol: $AB = \begin{bmatrix} 3 & 1 \\ 6 & 3 \end{bmatrix} \times \begin{bmatrix} 2 & 0 \\ 1 & 3 \end{bmatrix}$

$$= \begin{bmatrix} 3\times2 + 1\times1 & 3\times0 + 1\times3 \\ 6\times2 \quad 3\times1 & 6\times0 + 3\times3 \end{bmatrix}$$

$$\begin{bmatrix} 6 + 1 & 0 + 3 \\ 12 + 3 & 0 + 9 \end{bmatrix}$$

$$AB \begin{bmatrix} 7 & 3 \\ 15 & 9 \end{bmatrix}$$

$$BA = \begin{bmatrix} 2 & 0 \\ 1 & 3 \end{bmatrix} \times \begin{bmatrix} 3 & 1 \\ 6 & 3 \end{bmatrix}$$

$$\begin{bmatrix} 2\times3 + 0\times6 & 2\times1 + 0\times3 \\ 1\times3 + 3\times6 & 1\times1 + 3\times3 \end{bmatrix}$$

$$= \begin{bmatrix} 6 + 0 & 2 + 0 \\ 3 + 18 & 1 + 9 \end{bmatrix}$$

$$BA = \begin{bmatrix} 6 & 2 \\ 21 & 10 \end{bmatrix}$$

3. Find AB and BA $A = \begin{bmatrix} 6 & 0 \\ 1 & 7 \end{bmatrix}$ $B = \begin{bmatrix} -1 & 7 \\ 9 & 3 \end{bmatrix}$

Sol i) $AB \begin{bmatrix} 6 & 0 \\ 1 & 7 \end{bmatrix} \times \begin{bmatrix} -1 & 7 \\ 9 & 3 \end{bmatrix}$

$\begin{bmatrix} 6\times(-1) + 0\times 9 & 6\times 7 + 0\times 3 \\ 1\times(-1) + 7\times 9 & 1\times 7 + 7\times 3 \end{bmatrix}$

$= \begin{bmatrix} -6+0 & 42+0 \\ -1+63 & 7+21 \end{bmatrix}$

$AB = \begin{bmatrix} -6 & 42 \\ 62 & 28 \end{bmatrix}$

ii) $BA = \begin{bmatrix} -1 & 7 \\ 9 & 3 \end{bmatrix} \times \begin{bmatrix} 6 & 0 \\ 1 & 7 \end{bmatrix}$

$\begin{bmatrix} (-1)\times 6 + 7\times 1 & -1\times 0 + 7\times 7 \\ 9\times 6 + 3\times 1 & 9\times 0 + 3\times 7 \end{bmatrix}$

$BA = \begin{bmatrix} -6+7 & 0+49 \\ 54+3 & 0+21 \end{bmatrix}$

$BA = \begin{bmatrix} 1 & 49 \\ 57 & 21 \end{bmatrix}$

4. If $A = \begin{bmatrix} 1 & 6 \\ 2 & 3 \\ 5 & 1 \end{bmatrix}$ $B = \begin{bmatrix} 1 & 6 & 7 \\ 9 & 0 & -1 \end{bmatrix}$, find AB and BA

Sol $AB = \begin{bmatrix} 1 & 6 \\ 2 & 3 \\ 5 & 1 \end{bmatrix} \begin{bmatrix} 1 & 6 & 7 \\ 9 & 0 & -1 \end{bmatrix}$

$= \begin{bmatrix} 1+54 & 6+0 & 7-6 \\ 2+27 & 12+0 & 14-3 \\ 5+9 & 30+0 & 35-1 \end{bmatrix}$

$= \begin{bmatrix} 55 & 6 & 1 \\ 29 & 12 & 11 \\ 14 & 30 & 34 \end{bmatrix}$

$BA \begin{bmatrix} 1 & 6 & 7 \\ 9 & 0 & -1 \end{bmatrix} \begin{bmatrix} 1 & 6 \\ 2 & 3 \\ 5 & 1 \end{bmatrix}$

$= \begin{bmatrix} 1+12+35 & 6+18+7 \\ 9+0-5 & 54+0-1 \end{bmatrix}$

$= \begin{bmatrix} 48 & 31 \\ 4 & 53 \end{bmatrix}$

5. $A = \begin{bmatrix} 1 & 2 & -3 \\ 6 & 1 & 0 \\ 3 & 4 & 5 \end{bmatrix}$ $B = \begin{bmatrix} 1 & 2 & 3 \\ 1 & -1 & 1 \\ 7 & 8 & 9 \end{bmatrix}$ find AB.

$$AB = \begin{bmatrix} 1 & 2 & -3 \\ 6 & 1 & 0 \\ 3 & 4 & 5 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 1 & -1 & 1 \\ 7 & 8 & 9 \end{bmatrix}$$

$$= \begin{bmatrix} 1+2-21 & 2+2-24 & 3+2-27 \\ 6+1+0 & 12-1+0 & 18+1+0 \\ 3+4+35 & 6-4+40 & 9+4+45 \end{bmatrix}$$

$$= \begin{bmatrix} -18 & -24 & -22 \\ 7 & 11 & 19 \\ 42 & 42 & 58 \end{bmatrix}$$

6. If $A = \begin{bmatrix} 1 & 2 & -3 \\ 6 & 1 & 0 \\ 3 & 4 & 5 \end{bmatrix}$ $B = \begin{bmatrix} 1 & 2 & 3 \\ 1 & -1 & 1 \\ 7 & 8 & 9 \end{bmatrix}$ AB

$$AB \begin{bmatrix} 1 & 2 & -3 \\ 6 & 1 & 0 \\ 3 & 4 & 5 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 1 & -1 & 1 \\ 7 & 8 & 9 \end{bmatrix}$$

$$\begin{bmatrix} 1+2-21 & 2+2-24 & 3+2-27 \\ 6+1+0 & 12-1+0 & 18+1+0 \\ 3+4+35 & 6-4+40 & 9+4+45 \end{bmatrix}$$

$$\begin{bmatrix} -18 & -20 & -22 \\ 7 & 11 & 19 \\ 42 & 42 & 58 \end{bmatrix}$$

7. If $A = \begin{bmatrix} 1 & 3 \\ 4 & 2 \end{bmatrix}$ $B \begin{bmatrix} 2 & 1 \\ 4 & 4 \end{bmatrix}$ Show that $(AB)' = B'A'$

Let LHS $= (AB)'$

$$AB = \begin{bmatrix} 1 & 3 \\ 4 & 2 \end{bmatrix} \begin{bmatrix} 2 & 1 \\ 4 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 2+12 & 1+12 \\ 8+8 & 4+8 \end{bmatrix}$$

$$AB = \begin{bmatrix} 14 & 13 \\ 16 & 12 \end{bmatrix}$$

$$(AB)' = \begin{bmatrix} 14 & 13 \\ 16 & 12 \end{bmatrix} \rightarrow (1)$$

Let RHS $= B'A'$

$$B'A' = \begin{bmatrix} 2 & 4 \\ 1 & 4 \end{bmatrix} \begin{bmatrix} 1 & 4 \\ 3 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 2+12 & 8+8 \\ 1+12 & 4+8 \end{bmatrix}$$

$$= \begin{bmatrix} 14 & 16 \\ 13 & 12 \end{bmatrix} \rightarrow (2)$$

Eq (1) = Eq (2)

$\therefore$ LHS = RHS

$\therefore$ Hence Proved

8. If $A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$ Show that $A^2 - 4A - 5I = 0$

Let LHS $= A^2 - 4A - 5I$

$A^2 = A.A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$

$\begin{bmatrix} 1+4+4 & 2+2+4 & 2+4+2 \\ 2+2+4 & 4+1+4 & 4+2+2 \\ 2+4+2 & 4+2+2 & 4+4+1 \end{bmatrix}$

$A^2 = \begin{bmatrix} 9 & 8 & 8 \\ 8 & 9 & 8 \\ 8 & 8 & 9 \end{bmatrix}$

$4A = 4\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix} = \begin{bmatrix} 4 & 8 & 8 \\ 8 & 4 & 8 \\ 8 & 8 & 4 \end{bmatrix}$

$5I = 5\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix}$

LHS $= A^2 - 4A - 5I$

$= \begin{bmatrix} 9 & 8 & 8 \\ 8 & 9 & 8 \\ 8 & 8 & 9 \end{bmatrix} - \begin{bmatrix} 4 & 8 & 8 \\ 8 & 4 & 8 \\ 8 & 8 & 4 \end{bmatrix} - \begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix}$

$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = 0 =$ RHS

Hence proved.

9. If $A = \begin{bmatrix} 2 & 4 & 4 \\ 4 & 2 & 4 \\ 4 & 4 & 2 \end{bmatrix}$ then prove that $A^2 - 8A - 20I = 0$

Let LHS $= A^2 - 8A - 20I = 0$.

Sol. $A^2 = \begin{bmatrix} 2 & 4 & 4 \\ 4 & 2 & 4 \\ 4 & 4 & 2 \end{bmatrix} \begin{bmatrix} 2 & 4 & 4 \\ 4 & 2 & 4 \\ 4 & 4 & 2 \end{bmatrix}$

$= \begin{bmatrix} 4+16+16 & 8+8+16 & 8+16+8 \\ 8+8+16 & 16+4+16 & 16+8+8 \\ 8+16+8 & 16+8+8 & 16+16+4 \end{bmatrix}$

$A^2 = \begin{bmatrix} 36 & 32 & 32 \\ 32 & 36 & 32 \\ 32 & 32 & 36 \end{bmatrix}$

$8A = 8\begin{bmatrix} 2 & 4 & 4 \\ 4 & 2 & 4 \\ 4 & 4 & 2 \end{bmatrix}$

$8A = \begin{bmatrix} 16 & 32 & 32 \\ 32 & 16 & 32 \\ 32 & 32 & 16 \end{bmatrix}$

$20I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 20 & 0 & 0 \\ 0 & 20 & 0 \\ 0 & 0 & 20 \end{bmatrix}$

$\therefore$ LHS $= A^2 - 8A - 20I$

$\begin{bmatrix} 36 & 32 & 32 \\ 32 & 36 & 32 \\ 32 & 32 & 36 \end{bmatrix} - \begin{bmatrix} 16 & 32 & 32 \\ 32 & 16 & 32 \\ 32 & 32 & 16 \end{bmatrix} - \begin{bmatrix} 20 & 0 & 0 \\ 0 & 20 & 0 \\ 0 & 0 & 20 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

Hence proved

10. If $A = \begin{bmatrix} 2 & 1 & 1 \\ -1 & 0 & 1 \\ 1 & 3 & -1 \end{bmatrix}$ Calculate $A^2 - 5A + 9I$

$$A^2 = \begin{bmatrix} 2 & 1 & 1 \\ -1 & 0 & 1 \\ 1 & 3 & -1 \end{bmatrix} \begin{bmatrix} 2 & 1 & 1 \\ -1 & 0 & 1 \\ 1 & 3 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} 4+(-1)+1 & 2+0+3 & 2+1+1 \\ -2+0+1 & -1+0+3 & -1+0-1 \\ 2+(3)-1 & 1+0-3 & 1+3+1 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 4 & 5 & 2 \\ -1 & 2 & -2 \\ -2 & -2 & 5 \end{bmatrix}$$

$$5A = 5\begin{bmatrix} 2 & 1 & 1 \\ -1 & 0 & 1 \\ 1 & 3 & -1 \end{bmatrix} = \begin{bmatrix} 10 & 5 & 5 \\ -5 & 0 & 5 \\ 5 & 15 & -5 \end{bmatrix}$$

$$9I = 9\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 9 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 9 \end{bmatrix}$$

$$LHS = A^2 - 5A + 9I$$

$$\begin{bmatrix} 4 & 5 & 2 \\ -1 & 2 & -2 \\ -2 & -2 & 5 \end{bmatrix} - \begin{bmatrix} 10 & 5 & 5 \\ -5 & 0 & 5 \\ 5 & 15 & -5 \end{bmatrix} + \begin{bmatrix} 9 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 9 \end{bmatrix} = \begin{bmatrix} 3 & 0 & -3 \\ 4 & 11 & -7 \\ -7 & -17 & 19 \end{bmatrix}$$

∴ Hence proved.

Determinant :- A square matrix is said to be determinant if numbers are assigned unit along with row and columns within two parallel lines.

Ex: If $A = \begin{bmatrix} 3 & 2 \\ 1 & 4 \end{bmatrix}$ find determinant A

Sol: If $A = \begin{bmatrix} 3 & 2 \\ 1 & 4 \end{bmatrix}$

$$A = \begin{vmatrix} 3 & 2 \\ 1 & 4 \end{vmatrix}$$

$= 12 - 2$

$|A| = 10$

If $A = \begin{bmatrix} 6 & 7 \\ -8 & 3 \end{bmatrix}$

$|A| = \begin{vmatrix} 6 & 7 \\ 8 & 3 \end{vmatrix}$

$18 + 56$

$= 74$

Minors and cofactors :-

1. Find minors and cofactors of $A = \begin{bmatrix} 2 & 1 & 4 \\ 2 & 1 & 2 \\ 3 & 2 & 4 \end{bmatrix}$ $\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$

$M_{11} = \begin{vmatrix} 1 & 2 \\ 2 & 4 \end{vmatrix} = 4 - 4 = 0$ | $C_{11} = (-1)^{1+1} \times m_{11} = (-1)^2 \times 0 = 1 \times 0 =$

$M_{12} = \begin{vmatrix} 2 & 2 \\ 3 & 4 \end{vmatrix} = 8 - 6 = 2$ | $C_{12} = (-1)^{1+2} \times m_{12} = (-1)^3 \times 2 = -1 \times 2 =$

$M_{13} = \begin{vmatrix} 2 & 1 \\ 3 & 2 \end{vmatrix} = 4 - 3 = 1$ | $C_{13} = (-1)^{1+3} \times m_{13} = (-1)^4 \times 1 = 1 \times 1 =$

$M_{21} = \begin{vmatrix} 1 & 4 \\ 2 & 4 \end{vmatrix} = 4 - 8 = -4$ | $C_{21} = (-1)^{2+1} \times m_{21} = (-1)^3 \times -4 = -1 \times -4$

$M_{22} = \begin{vmatrix} 2 & 4 \\ 3 & 4 \end{vmatrix} = 8 - 12 = -4 \;/\; C_{22} = (-1)^{2+2} \times M_{22} = (-1)^4 \times -4 \ldots$

$M_{23} = \begin{vmatrix} 2 & 1 \\ 3 & 2 \end{vmatrix} = 4 - 3 = 1 \;/\; C_{23} = (-1)^{2+3} \times M_{23} = (-1)^5 \times 1 = -1\times \ldots$

$M_{31} = \begin{vmatrix} 1 & 4 \\ 1 & 2 \end{vmatrix} = 2 - 4 = -2 \;/\; C_{31} = (-1)^{3+1} \times M_{31} = (-1)^4 \times -2 = 1\times \ldots$

$M_{32} = \begin{vmatrix} 2 & 4 \\ 2 & 2 \end{vmatrix} = 4 - 8 = -4 \;/\; C_{32} = (-1)^{3+2} \times M_{32} = (-1)^5 \times -4 = -1\times -4 = \ldots$

$M_{33} = \begin{vmatrix} 2 & 1 \\ 2 & 1 \end{vmatrix} = 2 - 2 = 0 \;/\; C_{33} = (-1)^{3+3} \times M_{33} = (-1)^6 \times 0 = 1\times 0 = \ldots$

2. Find the minors and co-factors of $A = \begin{bmatrix} -1 & 0 & 2 \\ 1 & 3 & 4 \\ 1 & -1 & 6 \end{bmatrix}$

Sol, $\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} -1 & 0 & 2 \\ 1 & 3 & 4 \\ -1 & -1 & 6 \end{bmatrix}$

$M_{11} = \begin{vmatrix} 3 & 4 \\ -1 & 6 \end{vmatrix} = 18 - (-4) = 22 \;/\; C_{11} = (-1)^{1+1} \times M_{11} = (-1)^2 \times 22 = 1\times 22 = \ldots$

$M_{12} = \begin{vmatrix} 1 & 4 \\ 1 & 6 \end{vmatrix} = 6 - 4 = 2 \;/\; C_{12} = (-1)^{1+2} \times M_{12} = (-1)^3 \times 2 = -1\times 2 = \ldots$

$M_{13} = \begin{vmatrix} 1 & 3 \\ 1 & -1 \end{vmatrix} = -1 - 3 = -4 \;/\; C_{13} = (-1)^{1+3} \times M_{13} = (-1)^4 \times -4 = 1\times -4 = \ldots$

$M_{21} = \begin{vmatrix} 0 & 2 \\ -1 & 6 \end{vmatrix} = 0 - (-2) = 2 \;/\; C_{21} = (-1)^{2+3} \times M_{21} = -1 \times 2 = -2$

$M_{22} = \begin{vmatrix} -1 & 2 \\ 1 & 6 \end{vmatrix} = -6 - 2 = -8 \;/\; C_{22} = (-1)^{2+2} \times M_{22} = 1 \times -8 = -8$

$M_{23} = \begin{vmatrix} -1 & 0 \\ 1 & -1 \end{vmatrix} = 1 - 0 = 1 \;/\; C_{23} = (-1)^{2+3} \times M_{23} = -1 \times 1 = -1$

$M_{31} = \begin{vmatrix} 0 & 2 \\ 3 & 4 \end{vmatrix} = 0 - 6 = -6 \;/\; C_{31} = (-1)^{3+1} \times M_{31} = 1 \times -6 = -6$

$M_{32} = \begin{vmatrix} -1 & 2 \\ 1 & 4 \end{vmatrix} = -4 - 2 = -6 \;/\; C_{32} = (-1)^{3+2} \times M_{32} = -1 \times -6 = 6$

$M_{33} = \begin{vmatrix} -1 & 0 \\ 1 & 3 \end{vmatrix} = 3 - 0 = 3 \;/\; C_{33} = (-1)^{3+3} \times M_{33} = 1 \times 3 = -3$

I. Find Adjoint of a matrix

1. $A = \begin{bmatrix} 1 & -3 & 2 \\ 0 & 2 & 8 \\ -1 & 5 & 4 \end{bmatrix}$ find Adjoint. $\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$

Sol, $M_{11} = \begin{vmatrix} 2 & 8 \\ 5 & 4 \end{vmatrix} = 8 - 40 = -32 \;/\; C_{11} = (-1)^{1+1} \times M_{11} = (-1)^2 \times -32 = 1\times -32 = \ldots$

$M_{12} = \begin{vmatrix} 0 & 8 \\ -1 & 4 \end{vmatrix} = 0 - 8 = -8 \;/\; C_{12} = (-1)^{1+2} \times M_{12} = (-1)^3 \times -8 = -1\times -8 = \ldots$

$M_{13} = \begin{vmatrix} 0 & 2 \\ -1 & 5 \end{vmatrix} = 0 - 2 = -2 \;/\; C_{13} = (-1)^{1+3} \times M_{13} = (-1)^4 \times -2 = 1\times -2 = 2$

$M_{21} = \begin{vmatrix} -3 & 2 \\ 5 & 4 \end{vmatrix} = -12 - 10 = -22 \;/\; C_{21} = (-1)^{2+1} \times M_{21} = (-1)^3 \times -22 = -1\times -22 = \ldots$

$M_{22} = \begin{vmatrix} 1 & 2 \\ -1 & 4 \end{vmatrix} = 4 - (-2) = 6 \;/\; C_{22} = (-1)^{2+2} \times M_{22} = 1 \times 6 = 6$

$M_{23} = \begin{vmatrix} 1 & -3 \\ -1 & 5 \end{vmatrix} = 5-3 = 2 \;/\; C_{23} = (-1)^{2+3} \times M_{23} = -1 \times 2 = -2$

$M_{31} = \begin{vmatrix} -3 & 2 \\ 2 & 2 \end{vmatrix} = -24-4 = -28 \;/\; C_{31} = (-1)^{3+1} \times M_{31} = 1 \times -28 = -28$

$M_{32} = \begin{vmatrix} 1 & 2 \\ 0 & 8 \end{vmatrix} = 8-0 = 8 \;/\; C_{32} = (-1)^{3+2} \times M_{32} = -1 \times 8 = -8$

$M_{33} = \begin{vmatrix} 1 & -3 \\ 0 & 2 \end{vmatrix} = 2-0 = 2 \;/\; C_{33} = (-1)^{3+3} \times M_{33} = 1 \times 2 = 2$

$$\text{Adj } A = \begin{bmatrix} -32 & -8 & 2 \\ 22 & 6 & -2 \\ -28 & -8 & 2 \end{bmatrix}^{'} = \begin{bmatrix} -32 & 22 & -28 \\ -8 & 6 & -8 \\ 2 & -2 & 2 \end{bmatrix}$$

2. Find adjoint of $A = \begin{bmatrix} 1 & 2 & -3 \\ 6 & 0 & 3 \\ 2 & -1 & 1 \end{bmatrix}$

Sol $\Rightarrow A = \begin{bmatrix} 1 & 2 & -3 \\ 6 & 0 & 3 \\ 2 & -1 & 1 \end{bmatrix} \quad \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$

$M_{11} = \begin{vmatrix} 0 & 3 \\ -1 & 1 \end{vmatrix} = 0-(-3) = 3 \;/\; C_{11} = (-1)^{1+1} \times M_{11} = 1 \times 3 = 3$

$M_{12} = \begin{vmatrix} 6 & 3 \\ 2 & 1 \end{vmatrix} = 6-6 = 0 \;/\; C_{12} = (-1)^{1+2} \times M_{12} = -1 \times 0 = 0$

$M_{13} = \begin{vmatrix} 6 & 0 \\ 2 & -1 \end{vmatrix} = -6-0 = -6 \;/\; C_{13} = (-1)^{1+3} \times M_{13} = 1 \times -6 = -6$

$M_{21} = \begin{vmatrix} 2 & -3 \\ -1 & 1 \end{vmatrix} = 2-3 = -1 \;/\; C_{21} = (-1)^{2+1} \times M_{21} = -1 \times -1 = 1$

$M_{22} = \begin{vmatrix} 1 & -3 \\ 2 & 1 \end{vmatrix} = 1-(-6) = 7 \;/\; C_{22} = (-1)^{2+2} \times M_{22} = 1 \times 7 = 7$

$M_{23} = \begin{vmatrix} 1 & 2 \\ 2 & -1 \end{vmatrix} = -1-4 = -5 \;/\; C_{23} = (-1)^{2+3} \times M_{23} = -1 \times -5 = 5$

$M_{31} = \begin{vmatrix} 2 & -3 \\ 0 & 3 \end{vmatrix} = 6-0 = 6 \;/\; C_{31} = (-1)^{3+1} \times M_{31} = 1 \times 6 = 6$

$M_{32} = \begin{vmatrix} 1 & -3 \\ 6 & 3 \end{vmatrix} = 3-(-18) = 21 \;/\; C_{32} = (-1)^{3+2} \times M_{32} = -1 \times 21 = -$

$M_{33} = \begin{vmatrix} 1 & 2 \\ 6 & 0 \end{vmatrix} = 0-12 = -12 \;/\; C_{33} = (-1)^{3+3} \times M_{33} = 1 \times -12 = -1$

$$\text{Adj } A = \begin{bmatrix} 3 & 0 & -6 \\ 1 & 7 & 5 \\ 6 & -21 & -12 \end{bmatrix}^{'} = \begin{bmatrix} 3 & 1 & 6 \\ 0 & 7 & -21 \\ -6 & 5 & -12 \end{bmatrix}$$

Determinant of a square matrix

1. Find determinant of the matrix $A \begin{bmatrix} -5 & 7 \\ -2 & 3 \end{bmatrix}$

$A = \begin{bmatrix} -5 & 7 \\ -2 & 3 \end{bmatrix}$

$= -15 + 14$

$= -1$

2. Find determinant of matrix $A = \begin{bmatrix} 12 & 15 \\ 2 & 3 \end{bmatrix}$

$A = \begin{vmatrix} 12 & 15 \\ 2 & 3 \end{vmatrix}$

$36 - 30$

$= 6$

3. Find determinant of matrix $A = \begin{vmatrix} 2 & 1 & 4 \\ 2 & 1 & 2 \\ 3 & 2 & 4 \end{vmatrix} \quad \begin{bmatrix} + & - & + \\ - & + & - \\ + & - & + \end{bmatrix}$

$|A| = \begin{vmatrix} 2 & 1 & 4 \\ 2 & 1 & 2 \\ 3 & 2 & 4 \end{vmatrix}$

$$= +2\begin{vmatrix}1&2\\2&4\end{vmatrix} - 1\begin{vmatrix}2&2\\3&4\end{vmatrix} + 4\begin{vmatrix}2&1\\3&2\end{vmatrix}$$

$= 2(4-4) - 1(8-6) + 4(4-3)$

$= 2(0) - 1(2) + 4(1)$

$= 0 - 2 + 4$

$= 2$

4. Find $\begin{vmatrix}2&1&4\\2&1&2\\3&2&4\end{vmatrix}$ C_2 $\begin{vmatrix}+&-&+\\-&+&-\\+&-&+\end{vmatrix}$

$$= -1\begin{vmatrix}2&2\\3&4\end{vmatrix} + 1\begin{vmatrix}2&4\\3&4\end{vmatrix} - 2\begin{vmatrix}2&4\\2&2\end{vmatrix}$$

$-1(8-6) + 1(8-12) - 2(4-8)$

$= -1(2) + 1(-4) - 2(-4)$

$-2 + (-4) + 8$

$= -2 - 4 + 8$

$= 2$

5. Find determinat of $\begin{bmatrix}1&2&-1\\1&1&2\\2&-1&1\end{bmatrix}$ $\begin{bmatrix}+&-&+\\-&+&-\\+&-&+\end{bmatrix}$

$$+2\begin{vmatrix}2&-1\\1&2\end{vmatrix} + 1\begin{vmatrix}1&-1\\-1&2\end{vmatrix} + 1\begin{vmatrix}1&2\\-1&1\end{vmatrix}$$

$+2(4+1) + 1(2-1) + 1(1+2)$

$2(5) + 1(1) + 1(3)$

$10 + 1 + 3$

$= 14$

Singular Matrix: If determinant $[|A|]$ is equal to zero Then it is called singular matrix.

10M

Non-Singular matrix: If determinant $[|A| \neq 0]$ is not equal to zero. Then it is non-Singular matrix

★ Inverse of matrix: $A^{-1} = \frac{1}{|A|} \operatorname{adj} A$

1. Find Inverse of $A = \begin{bmatrix}3&1\\-1&2\end{bmatrix}$

Sol: We have $A^{-1} = \frac{1}{|A|} \operatorname{adj} A$

$|A| = \begin{vmatrix}3&1\\-1&2\end{vmatrix}$

$= (3\times2)(-1\times1)$

$= 6+1$

$= 7$

$|A| = 7 \neq 0$

$\therefore |A|$ is non singular matrix

$\therefore A^{-1}$ exist

-2-4-8

$\begin{bmatrix}3&1\\-1&2\end{bmatrix}$ $\operatorname{adj} A = \begin{bmatrix}2&-1\\1&3\end{bmatrix}$

$A^{-1} = \frac{1}{|A|} \operatorname{adj} A$

$= \frac{1}{7}\begin{bmatrix}2&-1\\1&3\end{bmatrix}$

$\begin{bmatrix}2\times\frac{1}{7}&-1\times\frac{1}{7}\\1\times\frac{1}{7}&3\times\frac{1}{7}\end{bmatrix} = A^{-1}\begin{bmatrix}2/7&-1/7\\1/7&3/7\end{bmatrix}$

2. Find inverse of $A = \begin{bmatrix} 2 & -4 \\ -3 & 5 \end{bmatrix}$

Sol. We have $A^{-1} = \frac{1}{|A|} \text{adj} A$

$|A| = \begin{vmatrix} 2 & -4 \\ -3 & 5 \end{vmatrix}$

$= 10 - 12$

$= -2$

$|A| = -2 \neq 0$

$\therefore |A|$ is non singular matrix

$\therefore A^{-1}$ exist

$\text{adj} A = \begin{bmatrix} 5 & +4 \\ +3 & 2 \end{bmatrix}$

$A^{-1} = \frac{1}{|A|} \text{adj} A$

$= \frac{1}{-2} \begin{vmatrix} 5 & +4 \\ +3 & 2 \end{vmatrix}$

$= \begin{bmatrix} 5 \times 1/2 & +4 \times 1/2 \\ +3 \times 1/2 & 2 \times 1/2 \end{bmatrix} = \begin{bmatrix} -5/2 & -2 \\ -3/2 & -1 \end{bmatrix}$

3. Find inverse of $\begin{bmatrix} 1 & 2 & -1 \\ -1 & 1 & 2 \\ 2 & -1 & 1 \end{bmatrix}$

Sol. let $A = \begin{bmatrix} 1 & 2 & -1 \\ -1 & 1 & 2 \\ 2 & -1 & 1 \end{bmatrix}$

We have $A^{-1} = \frac{1}{|A|} \text{adj} A$

$A = \begin{vmatrix} 1 & 2 & -1 \\ -1 & 1 & 2 \\ 2 & -1 & 1 \end{vmatrix}$ $\begin{bmatrix} + & - & + \\ - & + & - \\ + & - & + \end{bmatrix}$

$= +1 \begin{vmatrix} 1 & 2 \\ -1 & 1 \end{vmatrix} - 2 \begin{vmatrix} -1 & 2 \\ 2 & 1 \end{vmatrix} - 1 \begin{vmatrix} -1 & 1 \\ 2 & -1 \end{vmatrix}$

$= +(1+2) - 2(-1)$

$= 1(1+2) - 2(-1-4) - 1(1-2)$

$= 1(3) - 2(-5) - 1(-1)$

$= 3 + 10 + 1 = 14 \neq 0$

$\begin{vmatrix} 1 & 2 & -1 \\ -1 & 1 & 2 \\ 2 & -1 & 1 \end{vmatrix}$ $\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$

$M_{11} = \begin{vmatrix} 1 & 2 \\ -1 & 1 \end{vmatrix} = 1 + 2 = 3 \,/\, C_{11} = (-1)^{1+1} \times M_{11} = 1 \times 3 = 3$

$M_{12} = \begin{vmatrix} -1 & 2 \\ 2 & 1 \end{vmatrix} = -1 - 4 = -5 \,/\, C_{12} = (-1)^{1+2} \times M_{12} = -1 \times -5 = 5$

$M_{13} = \begin{vmatrix} -1 & 1 \\ 2 & -1 \end{vmatrix} = 1 - 2 = -1 \,/\, C_{13} = (-1)^{1+3} \times M_{13} = 1 \times -1 = -1$

$M_{21} = \begin{vmatrix} 2 & -1 \\ -1 & 1 \end{vmatrix} = 2 - 1 = 1 \,/\, C_{21} = (-1)^{2+1} \times M_{21} = -1 \times 3 = -1$

$M_{22} = \begin{vmatrix} 1 & -1 \\ 2 & 1 \end{vmatrix} = 1 + 2 = 3 \,/\, C_{22} = (-1)^{2+2} \times M_{22} = 1 \times 3 = 3$

$M_{23} = \begin{vmatrix} 1 & 2 \\ 2 & -1 \end{vmatrix} = -1 - 4 = -5 \,/\, M_{23} = (-1)^{2+3} \times M_{23} = -1 \times -5 = 5$

$M_{31} = \begin{vmatrix} 2 & -1 \\ 1 & 2 \end{vmatrix} = 4 + (1) = 5 \,/\, C_{31} = (-1)^{3+1} \times M_{31} = 1 \times 5 = 5$

$M_{32} = \begin{vmatrix} 1 & -1 \\ -1 & 2 \end{vmatrix} = 2 - 1 = 1 \,/\, C_{32} = (-1)^{3+2} \times M_{32} = -1 \times 1 = -1$

$M_{33} = \begin{vmatrix} 1 & 2 \\ -1 & 1 \end{vmatrix} = 1 - (-2) = 3 \,/\, C_{33} = (-1)^{3+3} \times M_{33} = 1 \times 3 = 3$

$A = \begin{bmatrix} 3 & -5 & -1 \\ -1 & 3 & 5 \\ 5 & -1 & 3 \end{bmatrix} \quad Adj\,A = \begin{bmatrix} 3 & -1 & 5 \\ -5 & 3 & -1 \\ -1 & 5 & 3 \end{bmatrix}$

$A^{-1} = \frac{1}{|A|} \text{adj}\, A$

$= \frac{1}{14} \begin{bmatrix} 3 & -1 & 5 \\ -5 & 3 & -1 \\ -1 & 5 & 3 \end{bmatrix}$

$= \begin{bmatrix} 3 \times 1/14 & -1 \times 1/14 & 5 \times 1/14 \\ -5 \times 1/14 & 3 \times 1/14 & -1 \times 1/14 \\ -1 \times 1/14 & 5 \times 1/14 & 3 \times 1/14 \end{bmatrix}$

$= \begin{bmatrix} 3/14 & -1/14 & 5/14 \\ -5/14 & 3/14 & -1/14 \\ -1/14 & 5/14 & 3/14 \end{bmatrix}$

4. Find the inverse of $\begin{bmatrix} 1 & -3 & 2 \\ 0 & 2 & 8 \\ -1 & 5 & 7 \end{bmatrix}$

Sol: let $A = \begin{bmatrix} 1 & -3 & 2 \\ 0 & 2 & 8 \\ -1 & 5 & 7 \end{bmatrix}$

we have $A^{-1} = \frac{1}{|A|} \text{adj}\, A$

$|A| = \begin{vmatrix} 1 & -3 & 2 \\ 0 & 2 & 8 \\ -1 & 5 & 7 \end{vmatrix} \quad \begin{vmatrix} + & - & + \\ - & + & - \\ + & - & + \end{vmatrix}$

$= +1 \begin{vmatrix} 2 & 8 \\ 5 & 7 \end{vmatrix} + 3 \begin{vmatrix} 0 & 8 \\ -1 & 7 \end{vmatrix} + 2 \begin{vmatrix} 0 & 2 \\ -1 & 5 \end{vmatrix}$

$= +1(14 - 40) + 3(0 + 8) + 2(0 + 2)$

$+1(-26) + 3(8) + 2(2)$

$= -26 + 24 + 4 = \cancel{2 \neq 0}\ 2 \neq 0$

A is not singular matrix

$= 2$

$M_{11} = \begin{vmatrix} 2 & 8 \\ 5 & 7 \end{vmatrix} = 14 - 40 = -26 \,/\, C_{11} = (-1)^{1+1} \times M_{11} = 1 \times -26 = -26$

$M_{12} = \begin{vmatrix} 0 & 8 \\ -1 & 7 \end{vmatrix} = 0 + 8 = 8 \,/\, C_{12} = (-1)^{1+2} \times M_{12} = -1 \times 8 = -8$

$M_{13} = \begin{vmatrix} 0 & 2 \\ -1 & 5 \end{vmatrix} = 0 + 2 = 2 \,/\, C_{13} = (-1)^{1+3} \times M_{13} = 1 \times 2 = 2$

$M_{21} = \begin{vmatrix} -3 & 2 \\ 5 & 7 \end{vmatrix} = -21 - 10 = -31 \,/\, C_{21} = (-1)^{2+1} \times M_{21} = -1 \times -31 = 31$

$M_{22} = \begin{vmatrix} 1 & 2 \\ -1 & 7 \end{vmatrix} = 7 + 2 = 9 \,/\, C_{22} = (-1)^{2+2} \times M_{22} = 1 \times 9 = 9$

$M_{23} = \begin{vmatrix} 1 & -3 \\ -1 & 5 \end{vmatrix} = 5-3 = 2 \;/\; C_{23} = (-1)^{2+3} \times M_{23} = -1 \times 2 = -2$

$M_{31} = \begin{vmatrix} -3 & 2 \\ 2 & 8 \end{vmatrix} = -24-4 = -28 \;/\; C_{31} = (-1)^{3+1} \times M_{31} = 1 \times -28 = -28$

$M_{32} = \begin{vmatrix} 1 & 2 \\ 0 & 8 \end{vmatrix} = 8-0 = 8 \;/\; C_{32} = (-1)^{3+2} \times M_{32} = -1 \times 8 = -8$

$M_{33} = \begin{vmatrix} 1 & -3 \\ 0 & 2 \end{vmatrix} = 2-0 = 2 \;/\; C_{33} = (-1)^{3+3} \times M_{33} = 1 \times 2 = 2$

$\text{adj } A = \begin{bmatrix} -26 & -8 & 2 \\ 31 & 9 & -2 \\ -28 & -8 & 2 \end{bmatrix}^{-1}$

$\text{adj } A = \begin{bmatrix} -26 & 31 & -28 \\ -8 & 9 & -8 \\ 2 & -2 & 2 \end{bmatrix}$

$A^{-1} = \frac{1}{|A|} \text{adj } A$

$= \frac{1}{|2|} \begin{bmatrix} -26 & 31 & -28 \\ -8 & 9 & -8 \\ 2 & -2 & 2 \end{bmatrix} = \begin{bmatrix} -26 \times \frac{1}{2} & 31 \times \frac{1}{2} & -28 \times \frac{1}{2} \\ -8 \times \frac{1}{2} & 9 \times \frac{1}{2} & -8 \times \frac{1}{2} \\ 2 \times \frac{1}{2} & -2 \times \frac{1}{2} & 2 \times \frac{1}{2} \end{bmatrix}$

$A^{-1} = \begin{bmatrix} -13 & 31/2 & -14 \\ -4 & 9/2 & -4 \\ 1 & -1 & 1 \end{bmatrix}$

5 Find the inverse $A = \begin{bmatrix} 1 & -3 & -1 \\ 2 & 4 & 5 \\ -3 & 2 & -2 \end{bmatrix}$

let $A = \begin{bmatrix} 1 & -3 & -1 \\ 2 & 4 & 5 \\ -3 & 2 & -2 \end{bmatrix}$ $\begin{bmatrix} + & - & + \\ - & + & - \\ + & - & + \end{bmatrix}$

$= +1 \begin{vmatrix} 4 & 5 \\ 2 & -2 \end{vmatrix} + 3 \begin{vmatrix} 2 & 5 \\ -3 & -2 \end{vmatrix} - 1 \begin{vmatrix} 2 & 4 \\ -3 & 2 \end{vmatrix}$

$= +1(-8-10) + 3(-4+15) - 1(4+12)$

$= +1(-18) + 3(11) - 16$

$= -4 + 33 - 16$

$= -1$

$= -1 \neq 0$

A is non singular matrix

A exist

$M_{11} = \begin{vmatrix} 4 & 5 \\ 2 & -2 \end{vmatrix} = -8-10 = -18 \;/\; C_{11} = (-1)^{1+1} \times M_{11} = 1 \times -18 = -18$

$M_{12} = \begin{vmatrix} 2 & 5 \\ -3 & -2 \end{vmatrix} = -4-15 = -19 \;/\; C_{12} = (-1)^{1+2} \times M_{12} = -1 \times -19 = 19$

$M_{13} = \begin{vmatrix} 2 & 4 \\ -3 & 2 \end{vmatrix} = 4-12 = -8 \;/\; C_{13} = (-1)^{1+3} \times M_{13} = 1 \times -8 = -8$

$M_{21} = \begin{vmatrix} -3 & -1 \\ 2 & -2 \end{vmatrix} = 6+2 = 8 \;/\; C_{21} = (-1)^{2+1} \times M_{21} = -1 \times 8 = -8$

$M_{22} = \begin{vmatrix} 1 & -1 \\ -3 & -2 \end{vmatrix} = -2-3 = 5 \;/\; C_{22} = (-1)^{2+2} \times M_{22} = 1 \times 5 = 5$

$M_{23} = \begin{vmatrix} 1 & -3 \\ -3 & 2 \end{vmatrix} = 2-9 = -7 \;/\; C_{23} = (-1)^{2+3} \times M_{23} = -1 \times -7 = 7$

$M_{31} = \begin{vmatrix} -3 & -1 \\ 4 & 5 \end{vmatrix} = -15+4 = -11 \;/\; C_{31} = (-1)^{3+1} \times M_{31} = 1 \times -11 = -11$

$M_{32} = \begin{vmatrix} 1 & -1 \\ 2 & 5 \end{vmatrix} = 5+2 = 7 \;/\; C_{32} = (-1)^{3+2} \times M_{32} = -1 \times 7 = -7$

$M_{32} = \begin{vmatrix} 1 & 3 \\ 2 & 4 \end{vmatrix} = 4+6 = 10$ / $C_{33} = (-1)^{3+3} \times M_{33} = 1 \times 10 = 10$

$$\text{adj}A = \begin{bmatrix} -18 & 19 & 8 \\ -8 & 5 & 7 \\ -11 & -7 & 10 \end{bmatrix}^{1}$$

$$\text{adj}A = \begin{bmatrix} -18 & -8 & -11 \\ 19 & 5 & -7 \\ -8 & 7 & 10 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|} \text{adj} A$$

$$= \frac{1}{-1} \begin{bmatrix} -18 & -9 & -11 \\ 19 & 5 & -7 \\ -8 & 7 & 10 \end{bmatrix} = \begin{bmatrix} -18 \times \frac{1}{-1} & -9 \times \frac{1}{-1} & -11 \times \frac{1}{-1} \\ 19 \times \frac{1}{-1} & 5 \times \frac{1}{-1} & -7 \times \frac{1}{-1} \\ -8 \times \frac{1}{-1} & 7 \times \frac{1}{-1} & 10 \times \frac{1}{-1} \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} 18 & 9 & 11 \\ -19 & -5 & 7 \\ 8 & -7 & -10 \end{bmatrix}$$

★ Cramer's Rule:

Consider the system of equation

$a_1x + b_1y = d_1$

$a_2x + b_2y = d_2$

then

$$\Delta(\text{delta}) = \begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}$$

$$\Delta x = \begin{vmatrix} d_1 & b_1 \\ d_2 & b_2 \end{vmatrix} \qquad \Delta y = \begin{vmatrix} a_1 & d_1 \\ a_2 & d_2 \end{vmatrix}$$

$$x = \frac{\Delta x}{\Delta}, \quad y = \frac{\Delta y}{\Delta}$$

1) Solve by Cramer's rule

$6x + 5y = 2$

$4x - 3y = 14$

$a_1 = 6 \quad a_2 = 4$

$b_1 = 5 \quad b_2 = -3$

$d_1 = 2 \quad d_2 = 14$

$$\Delta = \begin{vmatrix} 6 & 5 \\ 4 & -3 \end{vmatrix} = -18 - 20 = -38$$

$$\Delta x = \begin{vmatrix} d_1 & b_1 \\ d_2 & b_2 \end{vmatrix} = \begin{vmatrix} 2 & 5 \\ 14 & -3 \end{vmatrix} = -6 - 70, \quad \Delta x = -76$$

$$\Delta y = \begin{vmatrix} a_1 & d_1 \\ a_2 & d_2 \end{vmatrix} = \begin{vmatrix} 6 & 2 \\ 4 & 14 \end{vmatrix} = 84 - 8, \quad \Delta y = 76$$

$$x = \frac{\Delta x}{\Delta} = \frac{+76}{+38} = 2 \qquad y = \frac{\Delta y}{\Delta} = \frac{76}{38} = -2$$

2) Solve by Cramer's rule

$3x - 6y = 7$

$2x + 3y = 15$

$a_1 = 3 \quad a_2 = 2$

$b_1 = -6 \quad b_2 = 3$

$d_1 = 7 \quad d_2 = 15$

$$\Delta = \begin{vmatrix} a_1 & a_2 \\ b_1 & b_2 \end{vmatrix} = \begin{vmatrix} 3 & 2 \\ -6 & 3 \end{vmatrix} = 9 + 12 = 21$$

$$\Delta x = \begin{vmatrix} d_1 & b_1 \\ d_2 & b_2 \end{vmatrix} = \begin{vmatrix} 7 & -6 \\ 15 & 3 \end{vmatrix} = 21 + 90 = 111$$

$$\Delta_y \begin{vmatrix} a_1 & d_1 \\ a_2 & d_2 \end{vmatrix} = \begin{vmatrix} 3 & 7 \\ 2 & 15 \end{vmatrix} = 45-14 = 31$$

$$x = \frac{\Delta_x}{\Delta} = \frac{37}{21} = \frac{37}{21} \qquad y = \frac{\Delta_y}{\Delta} = \frac{31}{21}$$

★ Cramer's Rule by 3 equations

$$a_1x + b_1y + c_1z = d_1$$
$$a_2x + b_2y + c_2z = d_2$$
$$a_3x + b_3y + c_3z = d_3$$

$$\Delta = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} \qquad \Delta x \begin{vmatrix} d_1 & b_1 & c_1 \\ d_2 & b_2 & c_2 \\ d_3 & b_3 & c_3 \end{vmatrix}$$

$$\Delta y = \begin{vmatrix} a_1 & d_1 & c_1 \\ a_2 & d_2 & c_2 \\ a_3 & d_3 & c_3 \end{vmatrix} \qquad \Delta z = \begin{vmatrix} a_1 & b_1 & d_1 \\ a_2 & b_2 & d_2 \\ a_3 & b_3 & d_3 \end{vmatrix}$$

$$x = \frac{\Delta x}{\Delta} \quad y = \frac{\Delta y}{\Delta} \quad z = \frac{\Delta z}{\Delta}$$

1 Solve by Cramer's Rule

$$x + y + z = 11$$
$$2x - 6y - z = 0$$
$$3x + 4y + 2z = 0$$

$a_1 = 1 \quad a_2 = 2 \quad a_3 = 3$

$b_1 = 1 \quad b_2 = -6 \quad b_3 = 4$

$c_1 = 1 \quad c_2 = -1 \quad c_3 = 2$

$$\Delta = \begin{vmatrix} 1 & 1 & 1 \\ 2 & -6 & -1 \\ 3 & 4 & 2 \end{vmatrix} \qquad \begin{vmatrix} + & - & + \\ - & + & - \\ + & - & + \end{vmatrix}$$

$$= +1\begin{vmatrix} -6 & -1 \\ 4 & 2 \end{vmatrix} - 1\begin{vmatrix} 2 & -1 \\ 3 & 2 \end{vmatrix} + 1\begin{vmatrix} 2 & -6 \\ 3 & 4 \end{vmatrix}$$

$$= 1(-12+4) - 1(4+3) + 1(2+8)$$
$$= 1(-8) - 1(7) + 1(26)$$
$$= -8 - 7 + 26$$
$$= -15 + 26$$
$$\Delta = 11$$

$$\Delta x = \begin{vmatrix} d_1 & b_1 & c_1 \\ d_2 & b_2 & c_2 \\ d_3 & b_3 & c_3 \end{vmatrix} = \begin{vmatrix} 11 & 1 & 1 \\ 0 & -6 & -1 \\ 0 & 4 & 2 \end{vmatrix}$$

$$= 0\begin{vmatrix} 1 & 1 \\ -6 & -1 \end{vmatrix} - 4\begin{vmatrix} 11 & 1 \\ 0 & -1 \end{vmatrix} + 2\begin{vmatrix} 11 & 1 \\ 0 & -6 \end{vmatrix}$$

$$= 0(-1+6) - 4(-11-0) + 2(-66-0)$$
$$= 5 + 44 - 132$$
$$= -88$$

$$\Delta y = \begin{vmatrix} a_1 & d_1 & c_1 \\ a_2 & d_2 & c_2 \\ a_3 & d_3 & c_3 \end{vmatrix} \quad \begin{vmatrix} 1 & 11 & 1 \\ 2 & 0 & -1 \\ 3 & 0 & 2 \end{vmatrix}$$

$$= +1\begin{vmatrix} 0 & -1 \\ 0 & 2 \end{vmatrix} - 11\begin{vmatrix} 2 & -1 \\ 3 & 2 \end{vmatrix} + 1\begin{vmatrix} 2 & 0 \\ 3 & 0 \end{vmatrix}$$

$$= 1(0+0) - 11(4+3) + 1(0-0)$$
$$= -11 \times 7 = -77$$

$$\Delta z = \begin{vmatrix} a_1 & b_1 & d_1 \\ a_2 & b_2 & d_2 \\ a_3 & b_3 & d_3 \end{vmatrix} \quad \begin{vmatrix} 1 & 1 & 11 \\ 2 & -6 & 0 \\ 3 & 4 & 0 \end{vmatrix}$$

$$+1\begin{vmatrix} -6 & 0 \\ 4 & 0 \end{vmatrix} - 1\begin{vmatrix} 2 & 0 \\ 3 & 0 \end{vmatrix} + 11\begin{vmatrix} 2 & -6 \\ 3 & 4 \end{vmatrix}$$

$$1(0-0) - 1(0-0) + 11(8+15)$$
$$= 11 \times 26$$
$$= 286$$

$$x = \frac{\Delta x}{\Delta} = \frac{-88}{11} = -8$$

$$y = \frac{\Delta y}{\Delta} = \frac{-77}{11} = -7$$

$$z = \frac{\Delta z}{\Delta} = \frac{286}{11} = 26$$

2 Using Cramer's rule solve

$$x + 4y + z = 6$$
$$2x + 3y - z = 5$$
$$6x - 2y - 3z = -7$$

$a_1 = 1$
$a_2 = 2$
$a_3 = 6$
$b_1 = 1$
$b_2 = 3$
$b_3 = -2$
$c_1 = 1$
$c_2 = -1$
$c_3 = -3$
$d_1 = 6$
$d_2 = 5$
$d_3 = -7$

$$\Delta = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = \begin{vmatrix} 1 & 4 & 1 \\ 2 & 3 & -1 \\ 6 & -2 & -3 \end{vmatrix} \quad \begin{bmatrix} + & - & + \\ - & + & - \\ + & - & + \end{bmatrix}$$

$$= +1\begin{vmatrix} 3 & -1 \\ -2 & -3 \end{vmatrix} - 4\begin{vmatrix} 2 & -1 \\ 6 & -3 \end{vmatrix} + 1\begin{vmatrix} 2 & 3 \\ 6 & -2 \end{vmatrix}$$
$$= 1(-9-2) - 4(-6+6) + 1(-4-18)$$
$$= 1(-11) - 4(0) + 1(-22)$$
$$= -11 - 22 = -33$$

$$\Delta x = \begin{vmatrix} d_1 & b_1 & c_1 \\ d_2 & b_2 & c_2 \\ d_3 & b_3 & c_3 \end{vmatrix} = \begin{vmatrix} 6 & 4 & 1 \\ 5 & 3 & -1 \\ -7 & -2 & -3 \end{vmatrix}$$

$$+6\begin{vmatrix} 3 & -1 \\ -2 & -3 \end{vmatrix} - 4\begin{vmatrix} 5 & -1 \\ -7 & -3 \end{vmatrix} + 1\begin{vmatrix} 5 & 3 \\ -7 & -2 \end{vmatrix}$$
$$= 6(-9-2) - 4(-15-7) + 1(-10+21)$$
$$= 6(-11) - 4(-22) + 1(11)$$
$$= -33$$

$$\Delta y = \begin{vmatrix} a_1 & d_1 & c_1 \\ a_2 & d_2 & c_2 \\ a_3 & d_3 & c_3 \end{vmatrix} = \begin{vmatrix} 1 & 6 & 1 \\ 2 & 5 & -1 \\ 6 & -7 & -3 \end{vmatrix} \quad \begin{bmatrix} + & - & + \\ - & + & - \\ + & - & + \end{bmatrix}$$

$$= +1\begin{vmatrix} 5 & -1 \\ -7 & -3 \end{vmatrix} - 6\begin{vmatrix} 2 & -1 \\ 6 & -3 \end{vmatrix} + 1\begin{vmatrix} 2 & 5 \\ 6 & -7 \end{vmatrix}$$
$$= +1(-15-7) - 6(-6+6) + 1(-14-30)$$
$$= 1(-22) - 6(0) + 1(-14-30)$$
$$= -22 - 44 = -66$$

$$\Delta z = \begin{vmatrix} a_1 & b_1 & d_1 \\ a_2 & b_2 & d_2 \\ a_3 & b_3 & d_3 \end{vmatrix} = \begin{vmatrix} 1 & 4 & 6 \\ 2 & 3 & 5 \\ 6 & -2 & -7 \end{vmatrix}$$

$$+1\begin{vmatrix} 3 & 5 \\ -2 & -7 \end{vmatrix} - 4\begin{vmatrix} 2 & 5 \\ 6 & -7 \end{vmatrix} + 6\begin{vmatrix} 2 & 3 \\ 6 & -2 \end{vmatrix}$$
$$= 1(-21+10) - 4(-14-30) + 6(-4-18)$$
$$1(-11) - 4(-44) + 6(-22)$$
$$= -11 + 44 - 132$$
$$= 99$$

$$x = \frac{\Delta x}{\Delta} = \frac{-33}{-33} = 1$$

$$y = \frac{\Delta y}{\Delta} = \frac{-66}{-33} = 2$$

$$z = \frac{\Delta z}{\Delta} = \frac{-99}{-33} = 3$$

3. Solve by using Cramer's Rule

$2x + y - z = 3$
$x + y + z = 1$
$x - 2y - 3z = 4$

$a_1 = 2$
$a_2 = 1$
$a_3 = 1$
$b_1 = 1$
$b_2 = 1$
$b_3 = -2$
$c_1 = -1$
$c_2 = 1$
$c_3 = -3$
$d_1 = 3$
$d_2 = 1$
$d_3 = 4$

$$\Delta = \begin{vmatrix} 2 & 1 & -1 \\ 1 & 1 & 1 \\ 1 & -2 & -3 \end{vmatrix} \quad \begin{vmatrix} + & - & + \\ - & + & - \\ + & - & + \end{vmatrix}$$

$$+2\begin{vmatrix} 1 & 1 \\ -2 & -3 \end{vmatrix} - 1\begin{vmatrix} 1 & 1 \\ 1 & -3 \end{vmatrix} - 1\begin{vmatrix} 1 & 1 \\ 1 & -2 \end{vmatrix}$$

$= 2(-3+2) - 1(-3-1) - 1(-2-1)$
$= 2(-1) - 1(-4) - 1(-3)$
$= -2 + 4 + 3 = 5$

$$\Delta x = \begin{vmatrix} 3 & 1 & 1 \\ 1 & 1 & 1 \\ 4 & -2 & -3 \end{vmatrix} \quad \begin{vmatrix} + & - & + \\ - & + & - \\ + & - & + \end{vmatrix}$$

$$+3\begin{vmatrix} 1 & 1 \\ -2 & -3 \end{vmatrix} - 1\begin{vmatrix} 1 & 1 \\ 4 & -3 \end{vmatrix} - 1\begin{vmatrix} 1 & 1 \\ 4 & -2 \end{vmatrix}$$

$= 3(-3+2) - 1(-3-4) - 1(-2-4)$
$= 3(-1) - 1(-7) - 1(-6)$
$= -3 + 7 + 6 = 10$

$$\Delta y = \begin{vmatrix} 2 & 3 & -1 \\ 1 & 1 & 1 \\ 1 & 4 & -3 \end{vmatrix}$$

$$+2\begin{vmatrix} 1 & 1 \\ 4 & -3 \end{vmatrix} - 3\begin{vmatrix} 1 & 1 \\ 1 & -3 \end{vmatrix} - 1\begin{vmatrix} 1 & 1 \\ 1 & 4 \end{vmatrix}$$

$2(-3-4) - 3(-3-1) - 1(4-1)$
$2(-7) - 3(-4) - 1(3)$
$= -14 + 12 - 3$
$= -5$

$$\Delta z = \begin{vmatrix} 2 & 1 & 3 \\ 1 & 1 & 1 \\ 1 & -2 & 4 \end{vmatrix}$$

$$2\begin{vmatrix} 1 & 1 \\ -2 & 4 \end{vmatrix} - 1\begin{vmatrix} 1 & 1 \\ 1 & 4 \end{vmatrix} + 3\begin{vmatrix} 1 & 1 \\ 1 & -2 \end{vmatrix}$$

$= 2(4+2) - 1(4-1) + 3(-2-1)$
$= 2(6) - 1(3) + 3(-3)$
$= 12 - 3 - 9$
$= 0$

$$x = \frac{\Delta x}{\Delta} = \frac{10}{5} = 2$$

$$y = \frac{\Delta y}{\Delta} = \frac{-5}{5} = -1$$

$$z = \frac{\Delta z}{\Delta} = \frac{0}{5} = 0$$

Module 3 Commercial Arithmatic

Simple interest :- $= SI = \frac{PTR}{100}$ principle time rate of interest (PTR)

1. How much interest on ₹ 10000 for 7years and 5% per annum

Sol, P = ₹10000
T = 7years
R = 5% per annum

$SI = \frac{PTR}{100}$

$= \frac{10000 \times 7 \times 5}{100}$

$= 3500$

2. ₹ 2000 deposited in the bank for 2years at simple interest 6.5%. How much will be balance at the end of 2years

Sol, P = 2000
T = 2years
R = 6.5%

$SI = \frac{PTR}{100}$

$= \frac{2000 \times 2 \times 6.5}{100}$

SI = 260

A = P + I
2000 + 260
A = 2260

3. Find the simple interest ₹ 570.25 for 2 ½ years at 3.33% per annum and also find the amount at end 2½ years.

Sol, P = 570.25
T = 2½ 2½ = 5/2 = 2.5years
R = 3.33 %

$S.I = \frac{PTR}{100}$

$\frac{570.25 \times 2.5 \times 3.33\%}{100}$

= 47.47

A = P + I
570.25 + 47.47
A = 617.72

4. Find the simple interest on ₹ 10,000 for 2years 3months at rate of 6% per annum

Sol, P = 10,000
T = 2years 3month = 2year + $\frac{3}{12}$
= 2 + 0.25years
= 2.25years

R = 6%

Note- month = 12
days = 365

$$S.I = \frac{10,000 \times 2.25 \times 6\%}{100}$$

$= 1350$

5. Compute simple interest on ₹4000 at rate of $7\frac{1}{2}\%$, the sum is borrowed on 3rd oct 2010 and repaid on 15th Jan 2011

Sol, P = 4000
R = $7\frac{1}{2}\%$ = 7.5%
3rd oct 2010 15th Jan 2011
= 29 + 30 + 31 + 15 = 105 days = $\frac{105}{365}$ years
= 0.2876 years

$$SI = \frac{PTR}{100}$$

$$= \frac{4000 \times 0.2876 \times 7.5\%}{100}$$

$= 86.28$

6. Find the simple interest on ₹15,300 for 3 years 7 months and 73 days at rate of 5% per annum

Sol, P = 15,300
R = 5%
T = 3 years 7 months and 73 days = $3 + \frac{7}{12} + \frac{73}{365}$
= 3 + 0.5833 + 0.2
T = 3.7833 years.

$$SI = \frac{PTR}{100}$$

$$= \frac{15300 \times 3.7833 \times 5}{100}$$

SI = 2894.2245

7. Find the simple on ₹10,000 on 23rd may 2009 and repaid on 16th oct 2010 the rate of interest 6% per annum

Sol, P = 10,000
R = 6%
T = 23th may 2009 / 16th oct 2010
= 9 + 30 + 31 + 31 + 30 + 31 + 30 + 31 + 31 + 28 + 31 + 30 + 31
+ 30 + 31 + 31 + 30 + 16
T = 512 days
T = $\frac{512}{365}$ yrs
T = 1.4027 years

$$SI = \frac{PTR}{100}$$

$$= \frac{10,000 \times 1.4027 \times 6}{100}$$

$= 841.62$

8. Determine the sum of the money amounts to ₹855 3 years at rate of 4.5%.

Sol, A = 855
T = 3yrs
R = 4.5%
P = ?

$$A = P + I$$

$$855 = P + \frac{PTR}{100}$$

$$= P\left(1 + \frac{TR}{100}\right)$$

$$855 = P\left(1 + \frac{3 \times 4.5}{100}\right)$$

$$= P(1 + 0.135)$$

$$855 = P(1.135)$$

$$\frac{855}{1.135} = P$$

$$P = 753.30$$

9. Determine the principle that amount ₹10,000 in 5years at rate of 5% per annum.

Sol, A = 10,000
T 5years
R = 5%

$$A = P + I$$

$$10,000 = P + \frac{PTR}{100}$$

$$= P\left(1 + \frac{TR}{100}\right)$$

$$10000 = P\left(1 + \frac{5 \times 5}{100}\right)$$

$$P(1 + 0.25)$$

$$10,000 = P(1.25)$$

$$\frac{10000}{1.25} = P$$

$$P = 8000$$

10. A person deposits for 73 days and received ₹6350. find the rate of simple interest. Deposited amount 6250

Sol P = 6250
T = 73day $\frac{73}{365}$ T = 0.2yrs
A = 6350

$$SI = \frac{PTR}{100}$$

$$A = P + I$$

$$6350 = 6250 + I$$

$$6350 - 6250 = I$$

$$I = 100$$

$$SI = \frac{PTR}{100}$$

$$100 = \frac{6250 \times 0.2 \times R}{100}$$

$$100 \times 100 = 1250 \times R$$

$$\frac{10000}{1250} = R$$

R = 8% per annum

11. At what rate of simple interest a sum double itself in 6 years

Sol. R = ?

T = 6 years (ε)

let sum(P) be 1000

A = 2000

$SI = \frac{PTR}{100}$

A = P + I

2000 = 1000 + I

1000 = 2000 - I

1000 = I

$SI = \frac{PTR}{100}$

$1000 = \frac{1000 \times 6 \times R}{100}$

$1000 \times 100 = 6000 \times R$

$1,00,000 = 6000 \times R$

$\frac{100,000}{6000} = R$

R = 16.66% per annum

12. A person took a loan from ₹ 5,000 from 1-1-2010 and repaid on 31/12/2010 with amount 5625. Find the interest charged by lender

Sol. P = 5000

T = 1-1-2010 / 31/12/2010

T = 1 year

A = 5625

R = ?

A = P + I

5625 = 5000 + I

5625 - 5000 = I

I = 625

$SI = \frac{PTR}{100}$

$SI \times 100 = PTR$

$\frac{SI \times 100}{PT} = R$

$\frac{625 \times 100}{5000 \times 1} = R$

R = 12.5% per annum

13. At what time will ₹800 amounts to ₹896 at 6% per annum simple interest

Sol. P = 800

A = 896

R = 6% p.a.

T = ?

[A = P + I]

896 = 800 + I

896 - 800 = I

I = 96

$SI = \frac{PTR}{100}$

$SI \times 100 = PTR$

$\frac{SI \times 100}{PR} = T$

$\frac{96 \times 100}{800 \times 6} = T$

T = 2 years

Compound Interest :- $CI = P\left(1+\frac{R}{100}\right)^T - P$

Amount formula :- $A = P\left(1+\frac{R}{100}\right)^T$

1. Compute compound Interest on ₹ 5000 at 5% rate of Interest per annum for years.

Sol, $P = 5000$
$R = 5\%$ pa
$T = 3$ years
$CI = ?$

$CI = P\left(1+\frac{R}{100}\right)^T - P$

$= 5000\left(1+\frac{5}{100}\right)^3 - 5000$

$= 5000(1+0.05)^3 - 5000$
$= 5000(1.05)^3 - 5000$
$= 5000 \times 1.1576 - 5000$
$= 5788 - 5000$
$CI = 788$

2. Find the compound Interest at Amt ₹ 20,000 at 5% per annum for three 3years

Sol, $P = 20,000$
$T = 3$yr
$R = 5\%$
$CI = ?$
$A = ?$

$CI = P\left(1+\frac{R}{100}\right)^T - P$

$= 20,000\left(1+\frac{5}{100}\right)^3 - 20,000$

$= 20,000(1+0.05)^3 - 20,000$
$= 20,000(1.05)^3 - 20,000$
$= 20,000 \times 1.1576 - 20,000$
$= 23152 - 20,000$
$CI = 3152$

$A = P\left(1+\frac{R}{100}\right)^T$

$= 20,000\left(1+\frac{5}{100}\right)^3$

$= 20,000(1+0.05)^3$
$= 20,000(1.05)^3$
$= 20,000 \times 1.1576$
$A = 23152$

3. Find the compound Interest on ₹ 20,000 at 6% per annum for 4 years. What is the simple Interest some.

$P = 20,000$
$T = 4$ years
$R = 6\%$
$CI = ?$
$SI = ?$

$CI = P\left(1+\frac{R}{100}\right)^T - P$

$= 20,000\left(1+\frac{6}{100}\right)^4 - 20,000$

$= 20,000(1+0.06)^4 - 20,000$
$= 20,000(1.06)^4 - 20,000$
$= 20,000 \times 1.2624 - 20,000$
$= 25248 - 20000$
$CI = 5248$

$SI = \frac{PTR}{100}$

$$SI = \frac{20000 \times 4 \times 6}{100}$$

$SI = 4800$

4. Find the compound Interest on ₹ 20,000 for 4 years at 4% pa Payable half yearly. What will be the compound Interest if payable annualy

Sol, $P = 20{,}000$
$T = 4$ years
$R = 4\%$

CI for half yearly

$$CI = P\left(1 + \frac{r}{100}\right)^{T} - P$$

$$= 20{,}000\left(1 + \frac{2}{100}\right)^{8} - 20{,}000$$

$= 20{,}000\,(1+0.02)^8 - 20{,}000$
$= 20{,}000\,(1.02) - 20{,}000$
$= 20{,}000 \times 1.1716 - 20{,}000$
$= 23432 - 20{,}000$
$CI = 3432$

5. Find the compound interest on ₹ 2000 8% p.a for 2 years by compound quarterly

Sol, $P: 2000$
$R: 8\%$ pa $= 8/4 = 2\%$ p.a
$T = 2$ years
$= 2 \times 4$
$T = 8$ years

$$CI = P\left(1 + \frac{R}{100}\right)^{T} - P$$

$$= 2000\left(1 + \frac{2}{100}\right)^{8} - 2000$$

$= 2000\left(1 + 2/100\right)^8 - 2000$
$= 2000\,(1+0.02)^8 - 2000$
$= 2000(1.02)^8 - 2000$
$= 2000 \times 1.1716 - 2000$
$= 2343.2 - 2000$
$= 343.2$

6. What sum amounts to ₹ 8000 after 4 years at 5% pa. compound

Sol, $A = 8000$
$T = 4$ years
$R = 5\%$ pa
$P = ?$

$$A = P\left(1 + \frac{R}{100}\right)^{T}$$

$$8000 = P\left(1 + \frac{5}{100}\right)^{4}$$

$8000 = P(1+0.05)^4$
$8000 = P(1.05)$
$8000 = P(1.21155)$
$\frac{8000}{1.2155} = P$
$P = 6581.6536$

7. How many years will rupees ₹ 10,000 amount to ₹ 11,580 at 5% annum, compound

Sol, $P = 10{,}000$
$A = 11580$
$R = 5\%$ pa
$T = ?$

$$A = P\left(1 + \frac{R}{100}\right)^{T}$$

$$11580 = 10000\left(1+\frac{5}{100}\right)^T$$

$$11580 = 10000\left(1+0.05\right)^T$$

$$11580 = 10,000(1.05)^T$$

$$\frac{11580}{10,000} = (1.05)^T$$

$$1.158 = (1.05)^T$$

$$(1.05)^3 = (1.05)^T$$

$$T = 3$$

8. A man borrowed ₹12500 from a bank. at the figures be paid 13520 for settlement. What is the percentage rate of compound Interest charged by the bank.

Sol: P = 12500
T = 2 years
A = 13520
R = ?

$$A = P\left(1+\frac{R}{100}\right)^T$$

$$13520 = 12500\left(1+\frac{R}{100}\right)^2$$

$$\frac{13520}{12500} = \left(1+\frac{R}{100}\right)^2$$

$$= 1.086 = \left(1+\frac{R}{100}\right)^2$$

Taking squa both the sides

$$\sqrt{1.0816} = \sqrt{\left(1+\frac{R}{100}\right)^2}$$

$$1.04 = 1 + \frac{R}{100}$$

$$1.04 - 1 = \frac{R}{}$$

$$0.04 = \frac{R}{100}$$

$$0.04 \times 100 = R$$

$$R = 4\%\ p.a.$$

9. What annual rate of interest compound annualy doubles an investment in 6 years.

Sol: R = ?
T = 6
let assume P = 5000
Then amount A = 10,000

$$A = P\left(1+\frac{R}{100}\right)^T$$

$$10,000 = 5000\left(1+\frac{R}{100}\right)^6$$

$$\frac{10000}{5000} = \left(1+\frac{R}{100}\right)^6$$

$$2 = \left(1+\frac{R}{100}\right)^6$$

Taking squa both side $2 = \left(1+\frac{R}{100}\right)^6$

$$\sqrt{2} = \sqrt{\left(1+\frac{R}{100}\right)^{\cancel{6}}}$$

$$1.4142 = 1+\frac{R}{100}$$

$$1.4142 - 1 = \frac{R}{100}$$

$$0.4142 \times 100 = R$$

$$R = 41.42$$

Taking 6T on both side

$$6\sqrt{2} = 6\sqrt{\left(1+\frac{R}{100}\right)^6}$$

$$8.4852 = \left(1+\frac{R}{100}\right)$$

$$R = 748.52\%\ p.a.$$

Annuity:- It means a fixed periodical payment to its specific period of time.
Ex:- EMIs, loan, life insurance.
Immediate Annuity:- An annuity the payment of which is enforced immediately is called immediate annuity.
Ex: Rent payable Building immediately of entering into rent agreement either at the end or beginning of the month.

True discount:- It is the simple interest calculated on the present value of the bill it is called retaining discount.
It is calculated by the using the formula

$$\text{True discount} = \frac{PNR}{100} \text{ or } \frac{ANR}{100+NR}$$

Where A = Amt (or) face value of the bill
P = Present value of the bill
R = Rate of Interest
N = No. of days

Beginning of the year [Amount of an Annuity due]:-

$$A = \frac{a}{i}\left[(1+i)^n - 1\right](1+i)$$

End of the year (Immediate Annuity):-

$$A = \frac{a}{i}\left[(1+i)^n - 1\right]$$

Present value of Annuity:-

$$P = \frac{a}{i}\left[1 + \frac{1}{(1+i)^n}\right]$$

Present value of perpetual Annuity

$$P = \frac{a}{i}$$

where A = Amount of annuity
a = Payment per period
i = Rate of Interest
n = No. of years

1. Find the sum immediate Annuity consisting of 6th annual payment of ₹200. If rate of interest 5% per annum

Sol: n = 6 years
a = ₹200
i = 5% p.a
$= \frac{5}{100}$ i = 0.05

$$A = \frac{a}{i}\left[(1+i)^n - 1\right]$$

$$= \frac{200}{0.05}\left[(1+0.05)^6 - 1\right]$$

$$= 4000\left[(1.05)^6 - 1\right]$$

$$= 4000\left[1.34009 - 1\right]$$

$$= 4000\left[0.34009\right]$$

$$= 1360.36$$

2. Find the amount annuity with payment of ₹300 is made at the end of every half year for two years @ 10% compound half yearly.

Sol: a = ₹300
n = 2 years × 2 = 4
i = 10%
$= 10\% = 5\% = \frac{5}{100} = 0.05$

$$A = \frac{a}{i}\left[(1+i)^n - 1\right]$$

$$= \frac{300}{0.05}\left[(1+0.05)^4 - 1\right]$$

$$= 6000\left[(1.05)^4 - 1\right]$$

$$= 6000\left[1.2155 - 1\right]$$

$$= 6000\left[0.2155\right]$$

$$= 1269$$

3. Find the value of each instalment to be made by each year so that he has to pay ₹ 10,000 in next 4 years, if money is worth 9%.

Sol. $A = ₹\ 10,000$

$n = 4$ years

$i = 9\% = \frac{9}{100} = 0.09$

$$A = \frac{a}{i}\left[(1+i)^n - 1\right]$$

$$10,000 = \frac{a}{0.09}\left[(1+0.09)^4 - 1\right]$$

$$900 = a\left[(1.09)^4 - 1\right]$$

$$900 = a\left[1.4115 - 1\right]$$

$$900 = a\left[0.4115\right]$$

$$a = 2187.1202$$

2. Find the present value

i) The amount of ₹ 1000 payable at the begining of the period for 6 years interest @ 5% per annum components

Sol. $a = 1000$

$n = 6$

$i = 5\%$ pa $\frac{5}{100} = 0.05$

$$P = \frac{a}{i}\left[1 + \frac{1}{(1+i)^n}\right]$$

$$= \frac{1000}{0.05}\left[1 + \frac{1}{(1+0.05)^6}\right]$$

$$= 20000\left[1 + \frac{1}{(1.05)^6}\right]$$

$$= 20000\left[1 + \frac{1}{1.34009}\right]$$

$$= 20000\left[1 + 0.7462\right]$$

$$= 20,000\left[1.7462\right]$$

$$= 34924$$

$$A = \frac{a}{i}\left[(1+i)^n - 1\right](1+i)$$

$$= 20,000\left[(1+0.05)^6 - 1\right]\left[1+0.05\right]$$

$$= 20000\left[(1.05)^6 - 1\right](1.05)$$

$$= 20000(1.34009 - 1)(1.05)$$

$$= 20000\left[0.34009\right](1.05)$$

$$A = 7141.89$$

- Bill Discounting

True discount

$$\boxed{TD = \frac{ANR}{100+NR}} \quad (or) \quad TD = \frac{PNR}{100}$$

Banker's discount

$$\boxed{BD = \frac{ANR}{100}}$$

Where,

A - Amount (or) face value of the bill

N = Period of discounting

R - Rate of interest

Banker's gain

$$\boxed{BG = BD - TD}$$

$$BG = \frac{AN^2R^2}{100(100+NR)}$$

Present value = Face value - True discount

(or)

$$P = \frac{A \times 100}{100+NR} \quad (or) \quad P = \frac{TD \times 100}{NR}$$

Face value (or) Amount of bill

$$A = P + I$$

$$A = P\left(1 + \frac{NR}{100}\right) \quad (or) \quad A = \frac{BD \times TD}{BD - TD}$$

1. i) TD ii) BD iii) BG

on bill of ₹10,450 due 3 months hence 5% p.a.

Sol. i) $A = 10450$

$N = \frac{3}{12} = 0.25$

$R = 5\%$

$$TD = \frac{ANR}{100+NR}$$

$$= \frac{10450 \times 0.25 \times 5}{100 + 0.25 \times 5}$$ → First multiple then add with 100

$$= \frac{13062.5}{501.25} = \frac{13062.5}{101.25}$$

$$= 26.0593 \quad 129.0123$$

ii) $BD = \frac{ANR}{100} = \frac{10450 \times 0.25 \times 5}{100}$

$$= 130.625$$

iii) $BG = BD - TD$

$$= 130.625 - 129.0123$$

$$= 1.6127$$

Ratios and proportions :

→ If a and b are two quantities of same kind then $a:b$ (or) $\frac{a}{b}$ is called ratio.

→ Where a is called antecedent
b is called consequent

Types of ratio

i) Unit ratio :- A unit ratio is a ratio where the both numbers are same
Ex: 1:1 , 6:6

ii) Inverse ratio:- A inverse ratio is one which obtained by interchanging antecedent and consequent
Ex:- A:B = B:A then B:A is called inverse ratio

iii) Duplicate ratio: If a:b is the ratio then $a^2:b^2$ is called duplicate ratio
Ex:- → 2:3 = 4:9,
→ 3:7 = 9:49

iv) Triplicate ratio:- If a:b is the ratio then $a^3:b^3$ is called triplicate ratio
Ex: → $3^3:7^3$
27 : 343

v) Sub duplicate ratio: If a:b is the ratio then $\sqrt{a}:\sqrt{b}$ is called sub duplicate ratio

Ex: If a:b = 9:16
then $\sqrt{a}:\sqrt{b}$ = 3:4

vi) Sub triplicate ratio: If a:b is the ratio then $\sqrt[3]{a}:\sqrt[3]{b}$ is called sub triplicate ratio

Ex:- If a:b = 8:27
then $\sqrt[3]{a}:\sqrt[3]{b}$ = 2:3

vii) Compound Ratio:- If a:b, c:d, e:f then
$\frac{a}{b}, \frac{c}{d}, \frac{e}{f} = \frac{a \times c \times e}{b \times d \times f}$

Ex: If 1:3, 2:9, 4:7
then $\frac{1}{3}, \frac{2}{9}, \frac{4}{7} = \frac{1 \times 2 \times 4}{3 \times 9 \times 7} = \frac{8}{189}$

1. If a:b:c = 2:3:4 then find $\frac{a}{b}:\frac{b}{c}:\frac{c}{a}$
Sol: $\frac{a}{b}:\frac{b}{c}:\frac{c}{a} = \frac{2}{3}:\frac{3}{4}:\frac{4}{2}$ → LCM 3,4,2
$= \frac{8:9:24}{12}$ — $\frac{2}{3}\times 12$: $\frac{3}{4}\times 12$: $\frac{4}{2}\times 12$
= 8:9:24

2. If A:B = 2:3 and B:C = 4:5 find A:B:C
Sol: Given, A:B = 2:3 and B:C = 4:5

A : B : C
2 : 3 : [3]
[4] : 4 : 5
8 : 12 : 15

3. If $A:B = \frac{1}{2} : \frac{1}{3}$ $B:C = \frac{1}{2} : \frac{1}{3}$ then find $A:B:C$

sol. $A:B = \frac{1}{2} : \frac{1}{3}$ $\quad \frac{1}{2} \times 6^3 \; \frac{1}{3} \times 6^2$

$= 3:2$

$B:C = 3:2$

$A:B:C$
$3:2\,\boxed{}$
$\boxed{}:3:2$
$9:6:4$

Module 4 Business Statistics

Statistics is an art of changing numbers into information

Definition:

According to F.E. Croxton and D.J. Cowden Statistics maybe defined as science of collection, presentation, analyzing interpretation of numerical of data.

According to Spiegel Statistics is concerned with Scientific method for collecting organizing, summarizing, presenting and analyzing data as well as drawing valid conclusion and making decisions

Measure of central tendency: It is the single value which can be considered as representative of a set of observation and around which the observation can be considered as centered. It is called average

Types of central tendency:

* Arithematic mean
* Median
* Mode
* Geomatric mean
* Harmonic mean

1) Arithematic mean: $Mean(\bar{x}) = \frac{\text{Sum of the given observation}}{\text{total no. of observation}}$

i) Individual series: $\bar{x} = \frac{\Sigma x}{n}$

Where Σx = Sum of the values
n = Number of terms

1. Calculate mean from the following
Sl.no A B C D E F G H I J
Values - 125 128 132 135 140 148, 155, 157, 159, 161

Sol,

Sl.no	x
A	125
B	128
C	132
D	135
E	140
F	148
G	155
H	157
I	159
J	161
	1440

$\bar{x} = \frac{\Sigma x}{n}$

$= \frac{1440}{10}$

$= 144$

2. The monthly income of 10 families is given in rupee calculate the mean

Families	A	B	C	D	E	F	G	H	I	J
income	850	700	810	750	500	800	420	2500	2300	1500

Sol,

Families	income
A	850
B	700
C	810
D	750
E	500
F	800
G	420
H	2500
I	2300
J	1500
	11,160

$\bar{x} = \frac{\Sigma n}{n}$

$= \frac{11160}{10}$

1116

ii) Discrete series: Mean $\bar{x} = \frac{\Sigma fx}{N}$

f = frequency
x = values
N = Total no. of series frequency

1. Following are the marks obtained by the student in Stats. Calculate the mean

Sol,

Marks x	No. of student f	fx
35	3	105
40	8	320
45	12	540
50	9	450
55	4	220
60	7	420
65	15	975
70	10	700
75	10	750
80	7	560
85	5	425
90	3	270
95	2	190
	N=95	Σfx = 5925

$\bar{x} = \frac{\Sigma fx}{N}$

$= \frac{5925}{95}$

$\bar{x} = 62.3684$

iii) Continous Series: In continous series variables are represented by class interval each class interval has its own class frequency.

Types of continous series
i) Inclusive method
ii) Exclusive method

→ Inclusive method: Mean $(\bar{x}) = \frac{\Sigma fx}{N}$ and $x = \frac{\text{Upper limit} + \text{lower limit}}{2}$

1. Calculate the mean from the following data

Inclusive method

Production	8-12	13-17	18-22	23-27	28-32	33-37	38-42	43-47
No. of frequency	8	17	20	50	75	30	25	40

Sol

class interval	f	x	fx
8-12	8	10	80
13-17	17	15	255
18-22	20	20	400
23-27	50	25	1250
28-32	75	30	2250
33-37	30	35	1050
38-42	25	40	1000
43-47	40	45	1800
47-52	5	50	250
	N=270		Σfx = 8335

find of x

$x = \frac{U.L + L.L}{2}$

$\bar{x} = \frac{\Sigma fx}{N}$

$= \frac{8335}{270}$

$= 30.87$

2. Calculate arithmetic mean from the following data

Similar

Production	5-9	10-14	15-19	20-24	25-29	30-34	35-39
No. of factories	15	14	17	22	8	7	11

Sol

Production	No. of factories	x	fx
5-9	15	7	105
10-14	14	12	168
15-19	17	17	289
20-24	22	22	484
25-29	8	27	216
30-34	7	32	224
35-39	11	37	407
	N=94		Σfx = 1893

$x = \frac{UL + LL}{2}$

$\bar{x} = \frac{\Sigma fx}{N}$

$= \frac{1893}{94}$

$= 20.1382$

B Exclusive method:-

1. Calculate arithmetic mean from the following data

Production	10-20	20-30	30-40	40-50	50-60	60-70	70-80
No. of frequency	5	4	7	12	10	8	4

Total of items = n
Total no. of figures

Sol)

CI	f	x	fx
10-20	5	15	75
20-30	4	25	100
30-40	7	35	245
40-50	12	45	540
50-60	10	55	550
60-70	8	65	520
70-80	4	75	300
	N=50		Σfx=2330

$$\bar{x} = \frac{\Sigma fx}{N}$$

$$\frac{2330}{50}$$

$$= 46.6$$

2. Calculate the arithmetic mean from the following data

Marks	Below 20	30	40	50	60	70	80	90	100
No. of students	10	18	25	32	43	61	67	85	100

Sol)

CI	length cf	f	x	fx
10-20	10	10	15	150
20-30	18	8	25	200
30-40	25	7	35	245
40-50	32	7	45	315
50-60	43	11	55	605
60-70	61	18	65	1170
70-80	67	6	75	450
80-90	85	18	85	1530
90-100	100	15	95	1425
		N=100		6090

$$\bar{x} = \frac{\Sigma fx}{N}$$

$$= \frac{6090}{100}$$

$$= 60.90$$

3. Calculate the arithmetic mean for the following

Marks	Above 20	30	40	50	60	70	80	90
No. of students	100	95	87	62	43	25	13	2

Sol)

CI	more than cf	f	x	fx
20-30	100	5	25	125
30-40	95	8	35	280
40-50	87	25	45	1125
50-60	62	19	55	1045
60-70	43	18	65	1120
70-80	25	12	75	900
80-90	13	11	85	935
90-100	2	2	95	190
		N=100		Σfx=5770

$$\bar{x} = \frac{\Sigma fx}{N}$$

$$= \frac{5770}{100}$$

$$= 57.7$$

4. Calculate the arithmetic mean

Marks	10	20	30	40	50	60	70	80
Frequency	15	103	83	60	42	23	13	3

Sol:

CI	More than cf	f	x	fx
10-20	115	12	15	180
20-30	103	15	25	375
30-40	88	20	35	700
40-50	68	25	45	1125
50-60	43	20	55	1100
60-70	23	10	65	650
70-80	13	10	75	750
80-90	3	3	85	255
		N=115		Σfx=5135

$$\bar{x} = \frac{\Sigma fx}{N}$$

$$= \frac{5135}{115}$$

$$= 44.65$$

Median:- A median is the value of variable which divides the group into 2 equal parts. One part comprising all values greater and other all values lesser than median.

Individual Series:

$$M_o = \left(\frac{n+1}{2}\right)^{th} \text{term}$$

1. Determine the median from the following
25, 15, 23, 40, 27, 25, 23, 25, 20.

Sol: x
15
20
23
23
25
25
25
27
40

$$M_o = \left(\frac{n+1}{2}\right)^{th}$$

$$= \left(\frac{9+1}{2}\right)^{th}$$

$$\left(\frac{10}{2}\right)^{th}$$

$$= 5^{th} \text{ term}$$

$$M_o = 25$$

2. Determine the following median

Roll. No	Marks obtained	x
1	43	31
2	48	37
3	65	43
4	57	48
5	31	48
6	60	57
7	37	59
8	48	60
9	78	65
10	59	78

$$M_o = \left(\frac{n+1}{2}\right)^{th}$$

$$\left(\frac{10+1}{2}\right)^{th}$$

$$\left(\frac{11}{2}\right)^{th}$$

$$5.5^{th} \text{ term}$$

$$\frac{5^{th}+6^{th}}{2}$$

$$= \frac{48+57}{2}$$

$$= \frac{105}{2}$$

$$M_e = 52.5$$

Discrite Series:

$$M_e = \left(\frac{N+1}{2}\right)^{th} \text{term}$$

1. Calculate the median for the following

Size of item	4	6	8	10	12	14	16
frequency	2	4	5	3	2	1	4

Sol:

x	f	cf
4	2	2
6	4	6
8	5	11
10	3	14
12	2	16
14	1	17
16	4	21
	N = 21	

$$M_e = \left(\frac{N+1}{2}\right)^{th} \text{term}$$

$$\left(\frac{21+1}{2}\right)^{th}$$

$$= \left(\frac{22}{2}\right)^{th}$$

$$= 11^{th} \text{ term}$$

$$M_e = 8$$

2. Calculate the median from the following data

Marks	15	46	30	20	10
frequency	10	40	20	12	16

Sol:

Marks x	f	cf
15	10	10
46	40	50
30	20	70
20	12	82
10	16	98
	N = 98	

$$M_e = \left(\frac{N+1}{2}\right)^{th} \text{term}$$

$$= \left(\frac{98+1}{2}\right)^{th}$$

$$= \left(\frac{99}{2}\right)^{th}$$

$$= 49.5^{th}$$

$$M_e = 46$$

Continous series:

Step-1 :- $\left(\frac{N}{2}\right)^{th}$ term

Step-2 :- $M_e = L + \frac{\frac{N}{2} - cf}{f} \times i$

where M_e = Median

L = lower limit of the median class

cf = cummulative frequency of the class preceding median class

f = frequency of median class
N = Total no. of frequency
i = Size of the class interval

1. Calculate the median of the following

Length (in units)	0-20	20-40	40-60	60-80	80-100	100-120
No. of units	1	14	35	85	90	15

Sol:

C I	f	cf
0-20	1	1
20-40	14	15
40-60	35	50
60-80	85	135
80-100	90	225
100-120	15	240
	N = 240	

Step-1 - $(N/2)^{th}$ term

$= \left(\frac{240}{2}\right)$

$= 120^{th}$ term

Step-2: $Me = L + \frac{N/2 - cf}{f} \times i$

$= 60 + \frac{120-50}{85} \times 20$

$= 60 + \frac{70}{85} \times 20$

$= 60 + 16.47$

$= 76.47$

2. Calculate the median from the following

wages	1-5	6-10	11-15	16-20	21-25	26-30	31-35	36-40	41-45	46-50
frequency	5	7	9	15	12	10	9	8	4	2

Sol:

wages	C I	f	CF
1-5	0.5 - 5.5	5	5
6-10	5.5 - 10.5	7	12
11-15	10.5 - 15.5	9	21
16-20	15.5 - 20.5	15	36
21-25	20.5 - 25.5	12	48
26-30	25.5 - 30.5	10	58
31-35	30.5 - 35.5	9	67
36-40	35.5 - 40.5	8	75
41-45	40.5 - 45.5	4	79
46-50	45.5 - 50.5	2	81
-0.5 +0.5		N = 81	

$\frac{LL\ of\ seconding\ CI - UL\ of\ preceding}{2}$

$= \frac{41-40}{2} = \frac{1}{2} = 0.5$

Step 1 - $(N/2)^{th}$

$= 81/2^{th}$

$= 40.5^{th}$ term

$\frac{Lower\ limit\ seconding\ CI - UL\ of\ preceding}{2}$

$= \frac{41-40}{2} = \frac{1}{2} = 0.5$

Step-2: $Me = l + \left(\frac{N/2 - cf}{f}\right) \times i$

$= 20.5 + \left(\frac{40.5-36}{12}\right) \times 5$

$= 20.5 + \left(\frac{4.5}{12} \times 5\right)$

$= 20.5 + 1.875$

$= 22.375$

3. Calculate the median

less than	10	20	30	40	50	60	70	80
frequency	4	16	40	76	96	112	120	125

Sol,

CI	less than cf	f
0-10	4	4
10-20	16	12
20-30	40	24
30-40	76	36
40-50	96	20
50-60	112	16
60-70	120	8
70-80	125	5

N = 125

Step 1 $\left(\frac{N}{2}\right)^{th}$

$= \frac{125}{2}$

$= 62.5$

Step 2 : $L + \left(\frac{N/2 - cf}{f}\right) \times i$

$30 + \left(\frac{62.5 - 40}{36}\right) \times 10$

$\frac{31.25 - 40}{36}$

$30 + \frac{22.5}{36} \times 10$

$30 + 6.25$

$= 36.25$

2. Find the median

more than	10	20	30	40	50	60	70	80
frequency	115	103	88	68	43	23	13	3

Sol,

CI	more than cf	f	cf
10-20	115	12	12
20-30	103	15	27
30-40	88	20	47
40-50	68	25	72
50-60	43	20	92
60-70	23	10	102
70-80	13	10	112
80-90	3	3	115

N = 115

Step-1 :- $\left(\frac{N}{2}\right)^{th}$ term

$\frac{115}{2}^{th}$

57.5^{th}

Step-2 :- $Me = L + \left(\frac{N/2 - cf}{f}\right) \times i$

$= 40 + \left(\frac{57.5 - 47}{25}\right) \times 10$

$= 40 + \left(\frac{10.5}{25}\right) \times 10$

$= 40 + 4.2$

$= 44.2$

Mode :- It is defined as the value of variable which occurs most frequently distribution

* Individual series :- locate mode in the following data

7, 12, 18, 5, 9, 6, 10, 9, 4, 9, 9

Here 9 is repeated maximum number of times that 4 times. Therefore ∴ Mode (z) = 9

* Continuous series. Mode (z) = $L + \frac{f_1 - f_0}{(f_1 - f_0) + (f_1 - f_2)} \times c$

where, L = lower limit
f_1 = Highest frequency
f_0 = Preceding value of highest frequency
f_2 = Succeeding value of highest frequency
c = Size (or) Common difference

1. Determine mode for the following

Variables	frequency
28	10
30	12
32	16
34	14
36	10
38	8 highest value
40	17
42	3
44	4

Mode (z) = 40

1. Continuous Series :- Calculate the modal wages

Daily wages	20-25	25-30	30-35	35-40	40-45	45-50
No. of units	1	3	8	12	7	5

Sol: $= L + \frac{f_1 - f_0}{(f_1 - f_0) + (f_1 - f_2)} \times c$

$35 + \frac{12 - 8}{(12-8) + (12-7)}$

$35 + \frac{4}{4+5} \times 5$

$35 + \frac{4}{9} \times 5$

$35 + 20/9$

$35 + 2.22$

37.22

2. Calculate mode from the following data

x	10-19	20-29	30-39	40-49	50-59	60-69	70-79
F	5	12	22	25	14	10	8

Sol.

x	f	CI
10-19	5	9.5 - 19.5
20-29	12	19.5 - 29.5
20-39	22 - f_0	29.5 - 39.5
40-49	25 - f_1	39.5 - 49.5
50-59	14 - f_2	49.5 - 59.5
60-69	10	59.5 - 69.5
70-79	8	69.5 - 79.5

(-0.5) (+0.5)

$\frac{\text{l.l. d. succeeding c.I. u.l.}}{2}$

$= \frac{60-59}{2} = \frac{1}{2} = 0.5$

$$z = L + \frac{f_1 - f_0}{(f_1 - f_0) + (f_1 - f_2)} \times c$$

$$= 39.5 + \frac{25 - 22}{(25-22) + (25-14)} \times 10$$

$$= 39.5 + \frac{3}{14} \times 10$$

$$39.5 + \frac{30}{14}$$

$$= 39.5 + 2.14$$

$$z = 41.64$$

Dispersion :- The term dispersion is used in two sense 1- It means all the items in group are set of same values
2- dispersion is used indicate the major difference of size of the item.

Types of dispersion:-

1) Range (R)
2) Quartile Deviation (Q.D)
3) Mean deviation (M.D)
4) Standard Deviation (S.D)
5) Lorenz term

Range :- It is a major of dispersion in which difference between the highest and lowest values of the series.

$R = L - S$

$R = (\text{largest value} - \text{Smallest value})$

$$\text{coefficient of Range} = \frac{L - S}{L + S}$$

1. Compare the range and its coefficient of the series

Series	Variable								
I	20	22	24	29	27	26	29	21	25
II	60	64	68	66	67	63	69	62	60
III	91	92	93	94	95	96	97	98	99

Sol. I $R = L - S$
$= 29 - 20$
$R = 9$

Coefficient Range $= \frac{L-S}{L+S}$

$= \frac{29-20}{29+20}$

$= 9/49$

$= 0.18$

II $R = L - S$
$= 69 - 60$
$= 9$

Coef $R = \frac{L-S}{L+S}$

$\frac{69-60}{69+60}$

$\frac{9}{129} = 0.06$

III $R = L - S$
$= 99 - 91$
$= 8$

Coef $R = \frac{L-S}{L+S}$

$= \frac{99-91}{99+91}$

$= \frac{8}{190} = 0.04$

2) Quartile Deviation:

$QD = \frac{Q_3 - Q_1}{2}$

where Q_3 = Upper quartile
Q_1 = Lower quartile

Coef of QD $= \frac{Q_3 - Q_1}{Q_3 + Q_1}$

Individual Series : $Q_1 = \left(\frac{N+1}{4}\right)^{th}$ term

$Q_3 = \left(3\left(\frac{N+1}{4}\right)\right)^{th}$ term

Q 1 Calculate quartile deviation and it coefficient from following data

20 25 22 28 34 32 34 24 26 30

Sol.

x
20
22
24
25
26
28
30
32
34

$\boxed{QD = \frac{Q_3 - Q_1}{2}}$

$Q_1 = \left(\frac{N+1}{4}\right)^{th}$ term

$= \left(\frac{9+1}{4}\right)^{th}$

$= \left(10/4\right)^{th}$

$= 2.5$

$Q_1 = 2^{nd}$ term $+ 0.5(3^{rd}$ term $- 2^{nd}$ term$)$
$= 22 + 0.5(24 - 22)$
$= 22 + 0.5(2)$
$= 22 + 1$
$Q = 23$

$Q_3 = \left(3\left(\frac{N+1}{4}\right)\right)^{th}$ term

$= (3(2.5))^{th}$ term

$= (7.5)^{th}$ term

$Q_3 = 7^{th}$ term $+ 0.5(8^{th}_{term} - 7^{th}_{term})$

$= 30 + 0.5(32 - 30)$

$= 30 + 0.5(2)$

$= 30 + 1$

$Q_3 = 31$

$QD = \frac{31-23}{2} = \frac{8}{2} = 4$

Coef QD $= \frac{Q_3 - Q_1}{Q_3 + Q_1} = \frac{31-23}{31+23}$

$= \frac{8}{54}$

$= 0.1481$

Discret Series :-

1. Find QD and it co-efficent from the following data

values	frequency
125	8
126	4
130	3
132	10
134	7
138	9
140	4
145	9

Sol:

Values	f	cf
125	8	8
126	4	12
130	3	15
132	10	25
134	7	32
138	9	41
140	4	45
145	9	54
	N = 54	

$QD = \frac{Q_3 - Q_1}{2} = \frac{140-130}{2} = \frac{10}{2} = 5$

$Q_1 = \left(\frac{N+1}{4}\right)^{th}$ term

$= \left(\frac{54+1}{4}\right)^{th}$

$= \left(\frac{55}{4}\right)^{th}$

$= 13.75^{th}$ term

$Q_1 = 130$

$Q_3 = \left(3\left(\frac{N+1}{4}\right)\right)^{th}$ term

$= 3(13.75))^{th}$ term

$= 41.25^{th}$

$Q_3 = 140$

Coef of QD $= \frac{Q_3 - Q_1}{Q_3 + Q_1}$

$= \frac{140-130}{140+130}$

$= \frac{10}{270}$

$= 0.0370$

Continuous Series:

$$Q_1 = L + \left(\frac{N/4 - cf}{f}\right) \times c$$

$$Q_3 = L + \left(\frac{3N/4 - cf}{f}\right) \times c$$

1. Find QD and Coefficient from the following data

Age	20-25	25-30	30-35	35-40	40-45
No. of persons	70	80	180	150	20

Sol:

CI	f	cf
20-25	70	70
25-30	80	150
30-35	180	330
35-40	150	480
40-45	20	500
	N=500	

$$Q = L + \left(\frac{N/4 - cf}{f}\right) \times c$$

$$Q = \left(\frac{N}{4}\right)^{th} = \frac{500}{4}$$

$$= 125^{th} \text{ term}$$

$$Q = 25 + \left(\frac{125 - 70}{80}\right) \times 5$$

$$= 25 + \left(\frac{55}{80}\right) \times 5$$

$$= 25 + 3.4375$$

$$Q = 28.4375$$

$$Q_3 = \left(3\left(\frac{N}{4}\right)\right)^{th}$$

$$= (3(125))^{th}$$

$$= (375)^{th} \text{ term}$$

$$Q_3 = L + \left(\frac{3(N/4) - cf}{f}\right) \times c$$

$$= 35 + \frac{375 - 330}{150} \times 5$$

$$35 + \frac{45}{150_{30}} \times 5^1$$

$$= 35 + 1.5$$

$$Q_3 = 36.5$$

$$QD = \frac{Q_3 - Q_1}{2}$$

$$\frac{36.5 - 375}{2} \quad QD = \frac{36.5 - 28.435}{2}$$

$$= 4.03125$$

$$\text{coefficient of } QD = \frac{Q_3 - Q_1}{Q_3 + Q_1} = \frac{36.5 - 28.435}{36.5 + 28.435}$$

$$= \frac{8.065}{\ldots} = 0.1241$$

2. Calculate QD its coefficient from the following data

x	f
5 - 7	14
8 - 10	24
11 - 13	38
14 - 16	20
17 - 19	4

$\frac{\text{L.L of preceding} - \text{U.L of preceding}}{2}$

$= \frac{17-16}{2} = \frac{1}{2} = 0.5$

Sol,

x	f	cf
4.5 - 7.5	14	14
7.5 - 10.5	24	38
10.5 - 13.5	38	76
13.5 - 16.5	20	96
16.5 - 19.5	4	100
	N = 100	

$QD = \frac{Q_3 - Q_1}{2}$

$Q = L + \left(\frac{N/2 - cf}{f}\right) \times c$

$= (N/4)^{th} = \frac{100}{4}$

$= 25^{th}$ term

$= 7.5 + \left(\frac{25 - 14}{24}\right) \times 3$

$= 7.5 + \frac{11}{24} \times 3$

$= 7.5 + 1.375$

$= 8.875$

$Q_3 = L + \left(\frac{3(N/4) - cf}{f}\right) \times c$

$= 3(N/4)$

$= 3(25)$

$= 75$

$= 10.5 + \left(\frac{75 - 38}{38}\right) \times 3$

$= 10.5 + \frac{37}{38} \times 3$

$= 10.5 + \frac{111}{38}$

$= 10.5 + 2.9210$

$= 13.4210$

$QD = \frac{Q_3 - Q_1}{2} = \frac{13.4210 - 8.875}{2}$

$= \frac{4.546}{2}$

$= 2.273$

Coefficient of $QD = \frac{Q_3 - Q_1}{Q_3 + Q_1} = \frac{4.546}{22.296}$

$= 0.2039$

3. Mean deviation (MD) : $\frac{\Sigma d}{n}$

* Individual Series:

1. Find the mean deviation from the median and its coefficient

Sol:

x	$d = x - Me^{(42)}$
20	22
25	17
30	12
38	4
42	0
55	13
66	24
67	25
70	28
	$\Sigma d = 145$

$MD = \frac{\Sigma d}{n} = \frac{145}{9} = 16.11$

Coef of MD = $\frac{MD}{Median} = \frac{16.11}{42}$

$= 0.3835$

$d = x - Me$

1) $Me = \left(\frac{n+1}{2}\right)^{th}$ term

$= \left(\frac{9+1}{2}\right)^{th}$ term

$= \left(\frac{10}{2}\right)^{th}$ term

$= 5^{th}$ term

* Discrete Series:

$MD = \frac{\Sigma f d}{n}$

Coef = $\frac{MD}{median}$

2. Compute mean deviation and its coefficient from the median

Values	125	126	130	132	134	138	140	141
f	8	4	3	10	7	9	4	2

values	f	cf	$d = x - Me^{(132)}$	fd
125	8	8	7	56
126	4	12	6	24
130	3	15	2	6
132	10	25	0	0
134	7	32	2	14
138	9	41	6	54
140	4	45	8	32
141	2	47	9	18
	N: 47			Σfd : 204

$d = x - Me$

1) $Me = \left(\frac{N+1}{2}\right)^{th}$

$= \frac{47+1}{2}^{th}$

$= \frac{48}{2}^{th}$

$= 24^{th}$ term

$Me = 132$

$Md = \frac{\Sigma fd}{N}$

$= \frac{204}{47}$

$= 4.3404$

Coef of MD = $\frac{MD}{Median} = \frac{4.3404}{132} = 0.0328$

* Continuous series

1. Calculate Mean deviation and it coefficient from median

CI	0-10	10-20	20-30	30-40	40-50
F	8	30	40	12	10

CI	f	mid value	cf	$d = x - Me$	fd
0-10	8	5	8	18	144
10-20	30	15	38	8	240
20-30	40	25	78	2	80
30-40	12	35	90	12	144
40-50	10	45	100	22	220
	N=100				Σfd=828

$MD = \frac{\Sigma fd}{N} = \frac{828}{100} = 8.28$

Coefficient of MD $= \frac{MD}{median} = \frac{8.28}{23} = 0.36$

1) Step 1 $\left(\frac{N}{2}\right)^{th}$

$\left(\frac{100}{2}\right)^{th}$

50^{th}

Step 2 $Me = L + \left(\frac{\frac{N}{2} - cf}{f}\right) \times c$

$= 20 + \frac{50-38}{40} \times 10$

$= 20 + \frac{12}{40} \times 10$

$= 20 + 3$

$= 23$

Mode for mean deviation

1) Continuous Series

1. Calculate mean deviation and its co-efficient from mode.

CI	120-140	140-160	160-180	180-200	200-220	220-240	240-260	260-280
f	5	12	18	22	16	10	8	4

Soln

CI	f	mid value x	$d = x - z$	fd
120-140	5	130	58	290
140-160	12	150	38	456
160-180	18 f_0	170	18	324
180-200	22 f_1	190	2	44
200-220	16 f_2	210	22	352
220-240	10	230	42	420
240-260	8	250	62	496
260-280	4	270	82	328
	N=95			Σfd=2710

1) $Z = L + \left(\frac{f_1 - f_0}{(f_1 - f_0) + (f_1 - f_2)}\right) \times c$

$= 180 + \left(\frac{22 - 18}{(22-18) + (22-16)}\right) \times 20$

$= 180 + \frac{4}{10} \times 20$

$= 180 + 4 \times 2$

$= 180$

Mode = 188

2) $\bar{\delta} = \frac{\Sigma fd}{N}$

$= \frac{2710}{95}$

$= 28.5263$

3) Coeff of $\bar{\delta}$ by Mode Z

$= \frac{\bar{\delta}}{z}$

$= \frac{28.5263}{188} = 0.1517$

4. Standard deviation:-

$$\sigma = \sqrt{\frac{\Sigma d^2}{n}}$$

discrete of continuous series

$$\sigma = \sqrt{\frac{\Sigma fd^2}{N}}$$

Co-efficient of variation $= \frac{S.D.}{mean} \times 100$

* Individual Series:-

1) Calculate S.D and co-efficient of variation from following data

x	120	125	130	135	140	145	150	155	160	165

Sol

x	$d = x - \bar{x}$ (142.5)	d^2
120	-22.5	506.25
125	-17.5	306.25
130	12.5	156.25
135	-7.5	56.25
140	-2.5	6.25
145	2.5	6.25
150	12.5	56.25
155	12.5	156.25
160	17.5	306.25
165	22.5	506.25
$\Sigma x = 1425$		$\Sigma d^2 = 2062.5$

1) $\bar{x} = \frac{\Sigma x}{n} = \frac{1425}{10}$

$= 142.5$

2) $\sigma = \sqrt{\frac{\Sigma d^2}{n}}$

$= \sqrt{\frac{2062.5}{10}}$

$= \sqrt{206.25}$

$\sigma = 14.3614$

3) Co-efficient of variation $= \frac{\sigma}{\bar{x}} \times 100$

$= \frac{14.3614}{142.5} \times 100$

$= 0.10078 \times 100$

* discrete Series:-

1) Calculate SD and co-efficient from the following data.

wages	50	60	70	80	90	100	110	120
no of worker f	8	5	9	4	6	7	3	2

Sol

wages x	No. of worker f	fx	$d = x - \bar{x}$ (78.63)	d^2	fd^2
50	8	400	-28.63	819.67	6557.36
60	5	300	-18.63	347.07	1735.35
70	9	630	8.63	74.47	670.23
80	4	320	1.37	1.876	7.48
90	6	540	11.37	129.27	775.62
100	7	700	21.37	456.67	3196.69
110	3	330	31.37	984.07	2952.2
120	2	240	41.37	1711.47	3422.94
	$\Sigma f = 44$	$\Sigma fx = 3460$			$\Sigma fd^2 = 19317.89$

1) $\bar{x} = \frac{\Sigma fx}{N}$

$= \frac{3460}{44}$

$\bar{x} = 78.63$

2) $\sigma = \sqrt{\frac{\Sigma fd^2}{N}}$

$= \sqrt{\frac{19317.89}{44}}$

$= \sqrt{439.0427}$

$\sigma = 20.9533$

3) $COV = \frac{\sigma}{\bar{x}} \times 100$

$= \frac{20.9533}{78.63} \times 100$

$= 0.266 \times 100$

$= 26.6$

* Continous Series

1. Calculate S.D and its co-efficient from following data.

age	20-30	30-40	40-50	50-60	60-70	70-80	80-90
no. of marks	5	12	33	20	10	8	3

Sol,

age	No. of marks (f)	mid value (x)	fx	$d = x - \bar{x}$ (51)	d^2	fd^2
20-30	5	25	125	-26	676	3380
30-40	12	35	420	-16	256	3072
40-50	33	45	1485	-6	36	1188
50-60	20	55	1100	4	16	320
60-70	10	65	650	14	196	1960
70-80	8	75	600	24	576	4608
80-90	3	85	255	34	1156	3468
	$N = 91$		$\Sigma fx = 4635$			17,996

1) $\bar{x} = \dfrac{\Sigma fx}{N}$

$= \dfrac{4635}{91}$

$= 50.9 / 51$

$\bar{x} = 51$

2) $\sigma = \sqrt{\dfrac{\Sigma fd^2}{N}}$

$= \sqrt{\dfrac{17,996}{91}}$

$= \sqrt{197.25}$

$\sigma = 14.062$

3) $CoV = \dfrac{\sigma}{\bar{x}} \times 100$

$= \dfrac{14.062}{51} \times 100$

$= 27.57$

classmate
Date
Page

Correlation and regression Analysis

Correlation :- $R = \frac{\Sigma dx dy}{\sqrt{\Sigma dx^2 \Sigma dy^2}}$

where, $dx = x - \bar{x}$
$dy = y - \bar{y}$

1. Find the correlation coefficient by using karl pearson's method.

X	1	2	3	4	5
Y	4	5	6	7	8

Sol.

X	Y	$dx = x - \bar{x}$ (3)	$dy = Y - \bar{Y}$ (6)	$dxdy$	dx^2	dy^2
1	4	-2	-2	4	4	4
2	5	-1	-1	1	1	1
3	6	0	0	0	0	0
4	7	1	1	1	1	1
5	8	2	2	4	4	4
$\Sigma x = 15$	$\Sigma x = 30$			$\Sigma dxdy = 10$	$dx^2 = 10$	$dy^2 = 10$

$r = \frac{\Sigma dx\, dy}{\sqrt{\Sigma dx^2 \Sigma dy^2}}$

$= \frac{10}{} = \frac{10}{\sqrt{10 \times 10}}$

1) $\bar{x} = \frac{\Sigma x}{n} = \frac{15}{5} = 3$

2) $\bar{y} = \frac{\Sigma y}{n} = \frac{30}{5} = 6$

$= \frac{10}{\sqrt{100}}$

$= \frac{10}{10}$ $r = 1$

∴ Hence it is a perfect correlation

Probable error :- It is statistical measure which provide 2 limits which all the value obtain from different from sample point of the population.

$$P = 0.6745 \times \frac{1-r^2}{\sqrt{n}}$$

1. Calculate the coefficient of correlation from the following obb and calculate probable error

Marks in statistics	30	60	30	66	72	24	18	12	42	6
Marks in accounting	6	36	12	48	30	6	24	36	30	12

Sol

x	y	$dx = x - \bar{x}$ (36)	$dy = y - \bar{y}$ (24)	$dx\,dy$	dx^2	dy^2
30	6	-6	-18	108	36	324
60	36	24	12	288	576	144
30	12	-6	-12	72	36	144
66	48	30	24	720	900	576
72	30	36	6	216	1296	36
24	6	-12	-18	216	144	324
18	24	-18	0	0	324	0
12	36	-24	12	-288	576	144
42	30	6	6	36	39	36
6	12	-30	-12	360	900	144
$\Sigma x = 360$	$\Sigma y = 240$			$\Sigma dx dy = 1728$	4704	2232

1) $\bar{x} = \frac{\Sigma x}{n} = \frac{360}{10} = 36$

2) $\bar{y} = \frac{\Sigma y}{n} = \frac{240}{10} = 24$

$$r = \frac{\Sigma dx\,dy}{\sqrt{\Sigma dx^2\,\Sigma dy^2}}$$

$$= \frac{1728}{\sqrt{4224 \times 20242}}$$

$$= \frac{1728}{\sqrt{9030528}}$$

$$= \frac{1728}{3005.0836}$$

$$r = 0.5750$$

$\therefore$ There is a moderate degree positive correlation

$$PE = 0.6745 \times \frac{1-r^2}{\sqrt{n}}$$

$$P = 0.6745 \times \frac{1-(0.5750)^2}{\sqrt{10}}$$

$$= 0.6745 \times \frac{1-0.3306}{3.1622}$$

$$= 0.6745 \times \frac{0.6694}{3.1622}$$

$$PE = 0.1427$$

Spearman's Rank correlation :- If ranks are not repeated

$$R = 1 - \frac{6\Sigma d^2}{N^3 - N}$$

If ranks are repeated $R = 1 - \frac{6\left[\Sigma d^2 + \frac{1}{12}(m^3 - m) + \frac{1}{12}(m^3 - m) + \ldots\right]}{N^3 - N}$

Where $d = R_1 - R_2$

1. Two judges in a beauty competition ranked the 12 entries are as follows calculate rank correlation.

X	1	2	3	4	5	6	7	8	9	10	11	12
Y	12	9	6	10	3	5	4	7	8	2	11	1

Sol:

X	Y	$d = R_1 - R_2$	d^2
1	12	-11	121
2	9	-7	49
3	6	-3	9
4	10	-6	36
5	3	2	4
6	5	1	1
7	4	3	9
8	7	1	1
9	8	1	1
10	2	8	64
11	11	0	0
12	1	11	121
			$\Sigma d^2 = 416$

$$R = 1 - \frac{6\Sigma d^2}{N^3 - N}$$

where $d = R_1 - R_2$

$$= 1 - \frac{6 \times 416}{(12)^3 - 12}$$

$$= 1 - \frac{2496}{1728 - 12}$$

$$= 1 - \frac{2496}{1716}$$

$$= 1 - 1.4545$$

$$R = 0.4545$$

2. Calculate Rank correlation from the following data.

X	60	34	40	50	45	41	22	43	42	66
Y	75	32	35	40	45	33	45	50	45	40

Sol:

X	R_1	Y	R_2	$d = R_1 - R_2$	d^2
60	2	75	1	1	1
34	9	32	10	-1	1
40	8	35	8	0	0
50	3	40	6.5	-3.5	12.25
45	4	45	4	0	0
41	7	33	9	-2	4
22	10	45	4	6	36
43	5	50	2	3	9
42	6	45	4	2	4
66	1	40	6.5	-5.5	30.25
					$\Sigma d^2 = 97.5$

45 is repeated 3 times

$$\therefore \frac{3+4+5}{3} = \frac{12}{3} = 4^{th}$$

40 is repeated 2 times

$$\therefore \frac{6+7}{2} = \frac{13}{2} = 6.5$$

$$R = 1 - \frac{6\left[\Sigma d^2 + \frac{1}{12}(m^3 - m) + \frac{1}{12}(m^3 - m)\right]}{N^3 - N}$$

$$= 1 - \frac{6\left[97.5 + \frac{1}{12}((3)^3 - 3) + \frac{1}{12}((2)^3 - 2)\right]}{(10)^3 - 10}$$

$$\frac{1-6\left(97.5+\frac{1}{12}(7^3-7)+\frac{1}{12}(\ldots)\right)}{1000-10}$$

$$=1-6\frac{(97.5+2+0.5)}{990} = \frac{1-6(100)}{990}$$

$$=1-\frac{600}{990} = 1-0.6060 = 0.394$$

Regression:- The statistical technique that expresses the relation between 2 or more variables in the form of equation to estimate the value of variable based on the given value of another variable is called regression analysis.

Dependent Variable:- The variable whose value is estimated using the algebraic expression is called dependent variable.

Independent Variable:- The variable whose value is used to estimate another variable is called independent variable.

Regression
Lines of Regression

Regression line on 'X on Y'

$$x = a+by$$

where x = Dependent Variable
y = independent Variable
a and b = constant

Regression line on Y on X

$$y = a+bx$$

where x = Independent Variable
y = dependent Variable
a and b = constant

Regression Equation

X on Y	Y on X
$(x-\bar{x}) = bxy(y-\bar{y})$	$(y-\bar{y}) = byx(x-\bar{x})$

where bxy and byx are regression co-efficient

Method-1

When correlation coefficient, standard deviation (σ) of x and y is given

X on Y

$$(x-\bar{x}) = r\frac{\sigma x}{\sigma y}(y-\bar{y})$$

Y on X

$$(y-\bar{y}) = r\frac{\sigma y}{\sigma x}(x-\bar{x})$$

where $r = \sqrt{bxy\, byx}$

$$bxy = r\frac{\sigma x}{\sigma y}, \quad byx = r\frac{\sigma y}{\sigma x}$$

1. From the following information write 2 regression equation assume that the correlation co-efficient is perfectly positively

$\bar{x} = 46$
$\bar{y} = 39.5$
$\sigma_x = 4$
$\sigma_y = 5$
$r = 1$

Sol. X on Y

$(x-\bar{x}) = r\frac{\sigma_x}{\sigma_y}(y-\bar{y})$

$(x-46) = 1 \times \frac{4}{5}(Y-39.5)$

$(x-46) = 0.8(Y-39.5)$

$x - 46 = 0.8Y - 31.6$

$x = 0.8Y - 31.6 + 46$

$x = 0.8Y + 14.4$

Y on X

$(Y-\bar{Y}) = r\frac{\sigma_y}{\sigma_x}(x-\bar{x})$

$(Y-39.5) = 1 \times \frac{5}{4}(x-46)$

$(y-39.5) = \frac{5}{4}(x-46)$

$(y-39.5) = 1.25(x-46)$

$(y-39.5) = 1.25x - 57.5$

$y = 1.25x - 57.5 + 39.5$

$y = 1.25x - 18$

2. From the following equation
i) Write two regression equation
ii) Estimate the value of y when $x = 34$
iii) estimate the value of x when $y = 32$

Variables	Mean	S.D
X	48	6
Y	45	4
r	-0.54	

Sol. i) Given

$\bar{X} = 48,\ \bar{Y} = 45,\ \sigma_x = 6,\ \sigma_y = 4,\ r = -0.54$

X on Y

$(X-\bar{X}) = r\frac{\sigma_x}{\sigma_y}(y-\bar{y})$

$(x-48) = -0.54 \times \frac{6}{4}(Y-45)$

$(x-48) = \frac{3.24}{4}(y-45)$

$(x-48) = -0.81(y-45)$

$x = -0.81y + 36.45 + 48$

$x = -0.81y + 84.45$

Y on x

$(y-\bar{y}) = r\frac{\sigma_y}{\sigma_x}(x-\bar{x})$

$(y-45) = -0.54 \times \frac{4}{6}(x-48)$

$(y-45) = -0.36(x-48)$

$(y-45) = -0.36x + 17.28$

$y = -0.36x + 17.28 + 45$

$y = -0.36x + 62.28$

ii) $x = 34$

$y = -0.36x + 62.28$

$= -0.36(34) + 62.28$

$= -12.24 + 62.28 = 50.04$

iii) $y = 38$

$x = -0.81y + 84.45$

$x = -0.81(38) + 84.45$

$= -30.78 + 84.45$

$= 53.67$

* Method to solve the standard deviation of x and y are not given. The following formula should be applied

X on y

$(x-\bar{x}) = b_{xy}(y-\bar{y})$ where $b_{xy} = \frac{\Sigma xy}{\Sigma x^2}$

Y on x

$(y-\bar{y}) = b_{yx}(x-\bar{x})$ where $b_{xy} = \frac{\Sigma yx}{\Sigma x^2}$

1. The height in inches of a group of father and son are given below find the line of regression and estimate when father's height is 69 inches

X	Y	$x(x-\bar{x})$	$y(y-\bar{y})$	xy	y^2	x^2
71	69	2	4	8	16	4
68	64	-1	-1	1	1	1
66	65	-3	0	0	0	9
67	63	-2	-2	4	4	4
70	65	1	0	0	0	1
71	62	2	-3	-6	9	4
70	65	1	0	0	0	1
73	64	4	-1	4	1	16
72	66	3	1	3	1	9
65	69	-4	4	-16	16	16
66	62	-3	-3	9	9	9
$\Sigma x = 759$	$\Sigma y = 714$	18	0	$\Sigma xy = 7$	$y^2 = 52$	$x^2 = 74$

1) $\bar{x} = \frac{\Sigma x}{n} = \frac{759}{11}$

$= 69$

2) $\bar{y} = \frac{\Sigma y}{n} = \frac{714}{11}$

$= 64.909$

$= 65$ (Approx)

3) $b_{xy} = \frac{\Sigma xy}{\Sigma x^2}$

X on y

$(x-\bar{x}) = b_{xy}(y-\bar{y})$

$b_{xy} = \frac{\Sigma xy}{\Sigma y^2} = \frac{-1}{57}$

$(x-69) = \frac{-1}{57}(y-65)$

$(x-69) = -0.0175(y-65)$

$x = -0.0175y - 1.1025 + 69$

$x = -0.017y + 70.1375$

Y on x

$(y-\bar{y}) = b_{yx}(x-\bar{x})$ $\frac{\Sigma xy}{\Sigma x^2} = \frac{-1}{74}$

$(y-65) = -0.0133(x-69)$

$(y-65) = -0.0133x + 0.9177$

$y = -0.0133x + 65.9177$